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CONTENTS

Introduction. In order to appreciate the results obtained from mechanical tests of materials and to place upon them a proper interpretation, some knowledge of the elementary theory of elasticity is needful. The elastician views materials as of homogeneous structure and the usual engineering calculations are based on this view. Though materials, generally, are far from being homogeneous, the assumption nevertheless serves as a foundation for a vast body of analysis leading to results which the designer can apply with a considerable measure of success. The fundamental parts of the theory as understood by engineers will now be outlined.

Stress, Strain, Modulus of Elasticity. An external force, or load, applied to a body, produces therein an alteration of form. The deformation produced is commonly called the *strain,* while the resistance offered by the molecules of the body in an endeavour to preserve the original form is called *stress.* Quantitatively, *stress* is the load per unit area, tons per square inch or kilogrammes per square centimetre; while *strain* is the fractional alteration in dimensions, length, area or volume, as the case may be—or, in short, the change per unit dimension.

Consider a body in equilibrium under forces F_1, F_2, F_3 and F_4, (Fig. 1). Each small portion of the body will be in equilibrium under certain forces transmitted to its boundary. Imagine the section *ab*, of area A, to divide the body into two parts. The resultant R, of all the forces on one side of the section will be equal and opposite to the resultant of the forces on the other side of the section. Supposing the forces to be normal to and uniformly distributed over the section, the stress there will have the value R/A. If the forces are not uniformly distributed the stress will vary over the section and at any point its value

is defined as the ratio dR/dA, obtained by considering the force acting on the elemental area dA surrounding the point, the area being taken to be vanishingly small while the ratio dR/dA remains finite.

If, after the application and removal of the load, the strain disappears completely, the material is said to be perfectly elastic and the strain is then referred to as *elastic strain.* On the other hand, if the material be permanently deformed by the load, the strain is termed *plastic*, and the body is said to have received a *permanent set.*

When a body is strained by a steady load the strain increases until the stress induced just balances the applied force. A law enunciated by Hooke in 1576, states that, in an elastic body, the strain is proportional to the stress, so that doubling the applied force doubles the distortion produced in the body.

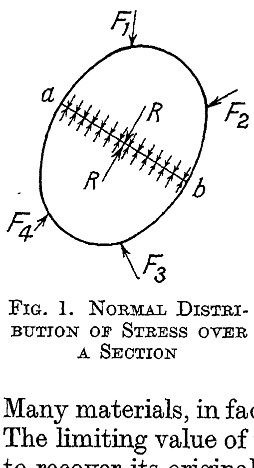


FIG. 1. NORMAL DISTRIBUTION OF STRESS OVER A SECTION

The ratio of the stress to the strain, termed the *modulus of elasticity*, varies for different materials and with the type of stressing applied.

All materials show a limit, the *limit of proportionality*, beyond which stress and strain cease to be proportional. Many materials, in fact, appear to possess no truly elastic range. The limiting value of the stress beyond which a material will fail to recover its original dimensions is termed the *elastic limit.*

Kinds of Stress. The state of stress suffered by a structural member or machine component is, in practice, brought about by a combination of several fundamental types of loading, namely—

(*a*) Tension;
(*b*) Compression;
(*c*) Shear;
(*d*) Torsion; and
(*e*) Flexure or bending. (Fig. 2.)

TENSION AND COMPRESSION. The forces applied constitute a pull or a push and tend to lengthen or shorten the body as the case may be. Extension is accompanied by a lateral contraction while compression is accompanied by lateral expansion.

If l be the initial length of a bar subjected to tension, and l_1 the length after the load has been applied, then $l_1 - l$ is the increase in length. The tensile strain e, or the extension per unit length, is therefore

$$e = (l_1 - l)/l$$

and is a pure ratio. It is assumed that the strain is uniform over the whole length of the rod.

If, for example, $l = 50$ in. and $l_1 = 50{\cdot}1$ in., the strain

$$e = \frac{50{\cdot}1 - 50}{50} = \frac{0{\cdot}1}{50} = 0{\cdot}002$$

In the same way, if d be the original diameter of the bar, supposed round, and d_1 the final diameter, the lateral strain is given by $(d_1 - d)/d$, and in this case is negative.

Similar results apply in compression.

Where tension or compression is concerned the modulus of elasticity is known as *Young's Modulus*; usually denoted by E.

By definition

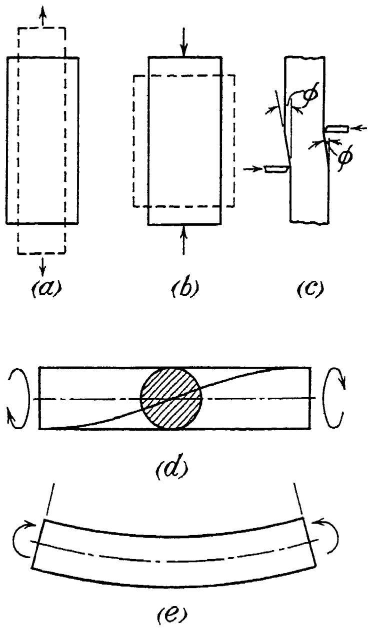


FIG. 2. TYPES OF LOADING

$$\text{Young's Modulus} = \frac{\text{stress}}{\text{strain}}$$

$$= \frac{\text{load/area}}{\text{extension/original length}}$$

$$= \mathrm{P}l/\mathrm{A}e\,.$$

EXAMPLE. A brass wire 0·04 in. diameter and 54 ft. long stretched 0·45 in. under a load of 12 lb.

The sectional area of the wire $= (0{\cdot}04)^2 \times (\pi/4) = 0{\cdot}001256$ in.^{2} The length $= 648$ in.

$$\text{Hence} \qquad \mathrm{E} = \frac{12 \times 648}{0{\cdot}001256 \times 0{\cdot}45} = 13\ 780\ 000 \text{ lb. per in.}^2$$

For bars whose section is other than circular the lateral strain is found by considering the difference between initial and final corresponding dimensions.

The ratio

$$\frac{\text{lateral strain}}{\text{longitudinal strain}} = \sigma$$

is termed *Poisson's Ratio*. It is sometimes defined by its reciprocal $m = 1/\sigma$. For steel m varies from 3 to 4 and in the absence of a more precise figure is frequently taken as 10/3.

SHEAR. The diagram Fig. 2 (c) shows equal and opposite forces acting on a bar at some distance apart. Such a system will

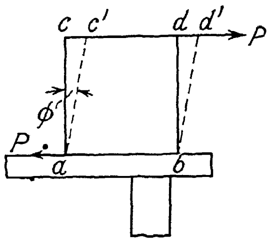


FIG. 3. ILLUSTRATING SHEAR STRAIN

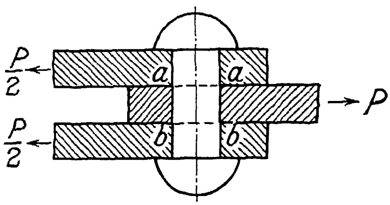


FIG. 4. RIVET IN DOUBLE SHEAR

produce a shear stress in the bar and is convenient for the purpose of illustration, but, in addition to the shear, bending stresses are set up and, in consequence, the member is not subjected to pure shear stress. The production of a pure shear stress in this way is difficult, if not impossible of attainment.

The characteristic feature of a shear is the angular change caused by the application of the load. The phenomenon is well illustrated by considering a block of indiarubber, Fig. 3, one face of which is glued to a table, while to the opposite face is glued a thin strip of wood. If the block of indiarubber is comparatively thin, bending effects may be neglected. The application of a pull P to the strip attached to the top face will result in an equal and opposite pull at the lower face and will distort the vertical faces ac and bd through an angle ϕ to the positions ac' and bd' respectively.

The shear strain is measured by the ratio $cc'/ac = \tan \phi$ and as cc' is small compared with ac, $\tan \phi$ may be replaced by ϕ radians. Shear strain is thus measured by the deviation from a right angle.

The ratio (shear stress)/(shear strain) is the *modulus of rigidity*, variously denoted in textbooks by C, G or N.

A practical example of shear stress is the riveted joint, Fig. 4, in which the rivet is in shear at the sections *aa* and *bb*—commonly termed *double shear*.

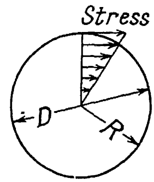


FIG. 5. VARIATION OF SHEAR STRESS IN CROSS-SECTION OF A ROUND SHAFT SUBJECTED TO TORSION

TORSION. Torsion is the shear produced when one layer of a body is made to rotate on the adjacent layer. A cylindrical shaft having equal and opposite torques or couples applied at its ends, and in which the axes of the couples coincide with the axis of the shaft, is subject at every section normal to the axis to pure shear stress. The stress at any point in a section is proportional to the distance of the point from the axis, being zero at the centre of the shaft and greatest at the extreme radius. (Fig. 5.) The external torque T applied, is balanced by the moment of resistance of the section, the relation being

$$T = (2\pi f_s/R) \times (R^4/4) = (\pi/16)D^3 f_s$$

where f_s is the shear stress at the extreme radius R.

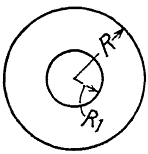


FIG. 6. CROSS-SECTION OF HOLLOW SHAFT

In the case of a hollow shaft of external diameter D and internal diameter D_1, Fig. 6, the relation becomes

$$\begin{aligned} T &= (2\pi f_s/R)(R^4/4 - R_1^4/4) \\ &= (\pi/16) f_s \,.\, [(D^4 - D_1^4)/D] \end{aligned}$$

For the solid shaft $\pi R^4/2$ is the polar moment of inertia J of the section; hence

applied torque $\qquad T = f_s(J/R) = f_s Z_p$

where $\qquad Z_p = J/R$

is termed the *modulus of the section in torsion*.

Imagine one end of the shaft to be fixed whilst the other is

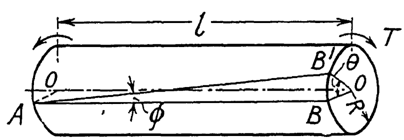


FIG. 7. ROUND SHAFT UNDER TORSIONAL STRAIN

subjected to a couple T, which twists the free end through an angle θ, a radius *OB* moving into the position *OB'*, Fig. 7. A

and as y is comparatively small in any practical case, y^2 may be neglected and the central deflection is given by

$$y = l^2/8R$$

It is shown in books on the strength of materials that

$$1/R = M/EI$$

where M is the bending moment, E is Young's modulus and I the moment of inertia of the section of the beam about the neutral axis. It follows that when the bending moment is the same at every section the central deflection of the beam is given by

$$y = Ml^2/8EI$$

In the general case of a beam loaded at right angles to its axis, as for example in the cantilever fixed at one end and loaded at the other, Fig. 9, any section of the beam has to sustain both the bending moment and a direct shear. The bending moment produced by the end load W, at any section distant x from the fixed end is $W(l - x)$ and reaches its greatest value at the point of fixing.

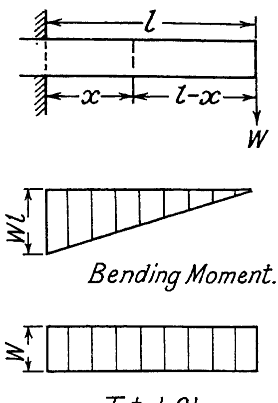


FIG. 9. CANTILEVER WITH END LOAD

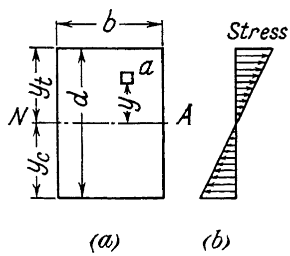


FIG. 10. FLEXURE

(a) Position of neutral axis. (b) Variation of stress over the section of a bent beam.

The shear stress is the same at every section and is proportional to W.

The upper fibres of the beam will be in tension and the lower fibres in compression. A particular layer somewhere in the beam will remain unstressed, so far as tension and compression are concerned. The plane of this layer is the *neutral surface* and the trace of this plane on a section perpendicular to the longitudinal axis of the beam, that is, the line NA, Fig. 10 (a), is called the *neutral axis* of the section.

The stress at any element of area a is proportional to its distance y from the neutral axis and the state of stress over the section is that represented in Fig. 10 (*b*), the material above NA being in tension and that below NA being in compression.

The neutral axis ordinarily passes through the centroid of the section. This, however, is merely fortuitous and is the outcome of the linearity of the stress distribution over the depth of the section and the fact that the loads act perpendicularly to the length of the beam. For a non-linear stress distribution, or for an initially curved beam, or for a straight beam subjected

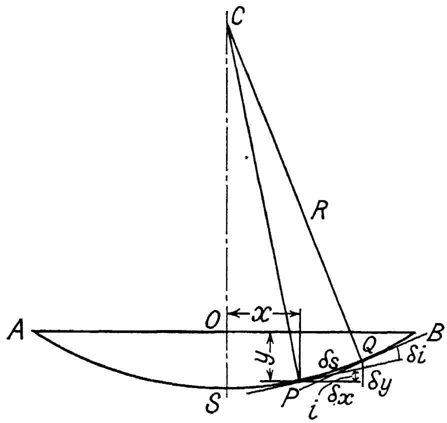


FIG. 11. CHANGE OF SLOPE IN A DEFLECTED BEAM

to oblique loading the position of the neutral axis is not necessarily coincident with the c.g. of the section.

Textbooks show that if y be the distance of an element from the neutral axis, the stress f there is given by

$$f/y = \mathrm{E}/\mathrm{R} = \mathrm{M}/\mathrm{I}$$

The moment of inertia of the section divided by the distance of the extreme fibre from the neutral axis is termed the *section modulus* Z. The maximum stress occurs at the outermost fibres, and, in general, will have different values in tension and compression.

If f_t and f_c are the respective tensile and compressive stresses at distances y_t and y_c from the neutral surface then, in tension

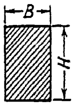
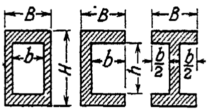
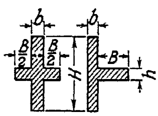
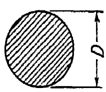
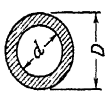
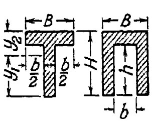
$$\mathrm{M} = f_t\mathrm{I}/y_t = f_t\mathrm{Z}_t$$

and in compression

$$\mathrm{M} = f_c\mathrm{I}/y_c = f_c\mathrm{Z}_c$$

TABLE I

GEOMETRICAL PROPERTIES OF SECTIONS

Section	Area In.^{2}	Moment of Inertia In.^{4}	Modulus of Section In.^{3}
		I	Z
	BH	$\frac{1}{12}BH^3$	$\frac{1}{6}BH^2$
	$BH - bh$	$\frac{1}{12}(BH^3 - bh^3)$	$\frac{BH^3 - bh^3}{6H}$
	$bH + Bh$	$\frac{1}{12}(bH^3 + Bh^3)$	$\frac{bH^3 + Bh^3}{6H}$
	$\pi\frac{D^2}{4}$	$\frac{\pi}{64}D^4$	$\frac{\pi}{32}D^3$
	$\frac{\pi}{4}(D^2 - d^2)$	$\frac{\pi}{64}(D^4 - d^4)$	$\frac{\pi}{32}\left(\frac{D^4 - d^4}{D}\right)$
	$BH - bh$	$I = \frac{(BH^2 - bh^2)^2 - 4BHbh(H - h)^2}{12(BH - bh)}$ $y_1 = \frac{BH^2 - bh^2}{2(BH - bh)}$ $y_2 = \frac{BH^2 - 2bhH + bh^2}{2(BH - bh)}$	$Z_1 = \frac{I}{y_1}$ $Z_2 = \frac{I}{y_2}$

where Z_t and Z_c are the section moduli in tension and compression respectively. Values of I and Z for a few standard sections are given in Table I. When the section is symmetrical about the neutral axis Z_c and Z_t have the same value.

Consider two near points on the neutral surface of a deflected beam at which the radius of curvature is R, Fig. 11. The slope of the beam at P, or its angular deviation from the horizontal there, is i and at Q the slope is $i + di$.

The change of slope from P to Q is thus equal to di, that is to

$$PQ/\mathrm{R} = ds/\mathrm{R} = dx/\mathrm{R}$$

for small deflections.

Further, $i = dy/dx$ and hence the curvature $1/\mathrm{R} = di/dx = (d/dx)(i) = (d/dx)(dy/dx) = (d^2y/dx^2)$. Hence, in terms of M, E and I,

$$d^2y/dx^2 = \mathrm{M/EI}$$

In general, M is a function of the distance x along the beam from some assigned origin and when so expressed in the foregoing equation one integration will enable the slope, and a second integration will enable the deflection to be found for any point along the beam.

The cantilever previously considered provides a simple example.

Here $\mathrm{M} = \mathrm{W}(l - x)$ so that

$$d^2y/dx^2 = \mathrm{W}(l - x)/\mathrm{EI}$$

therefore

$$i = dy/dx = (\mathrm{W/EI})(lx - x^2/2) + \mathrm{C}$$

where C is the constant of integration.

The slope is zero at the fixed end where $x = 0$, hence C is zero.

A second integration gives

$$y = (\mathrm{W/EI})(lx^2/2 - x^3/6) + \mathrm{D}$$

The deflection y is zero at the fixed end and therefore D is zero. The deflection at any point is therefore

$$y = (\mathrm{W/EI})(lx^2/2 - x^3/6)$$

and the maximum deflection δ, which is chiefly what is required, occurs at the free end where $x = l$, so

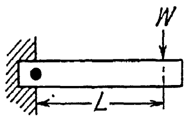
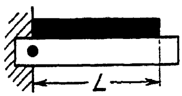
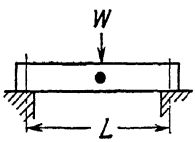
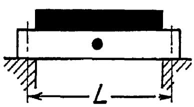
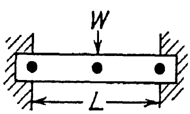
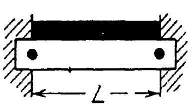
$$\delta = (\mathrm{W/EI})(l^3/2 - l^3/6) = \mathrm{W}l^3/3\mathrm{EI}$$

Some standard cases are given in Table II.

TABLE II

MAXIMUM BENDING MOMENT AND MAXIMUM DEFLECTION OF LOADED BEAMS

W = total load. w = load per inch run.

Type of Loading	Maximum Bending Moment	Maximum Deflection
	(occurs at ●)	
	WL	$\frac{WL^3}{3EI}$
	$\frac{1}{2}WL$ or $\frac{1}{2}wL^2$	$\frac{WL^3}{8EI}$
	$\frac{1}{4}WL$	$\frac{WL^3}{48EI}$
	$\frac{1}{8}WL$ or $\frac{1}{8}wL^2$	$\frac{5}{8}\cdot\frac{WL^3}{48EI}$
	$\frac{1}{8}WL$	$\frac{1}{4}\cdot\frac{WL^3}{48EI}$
	$\frac{1}{12}WL$ or $\frac{1}{12}wL^2$	$\frac{1}{8}\cdot\frac{WL^3}{48EI}$

Bulk Modulus of Elasticity. In addition to Young's modulus (E) and the modulus of rigidity (N), materials possess a volume modulus or *bulk modulus* (K) which represents the ratio between the change of pressure and the change of volume when the material is subjected to a uniform distribution of tension or compression at its outer boundary, as for example a body subjected to hydrostatic pressure.

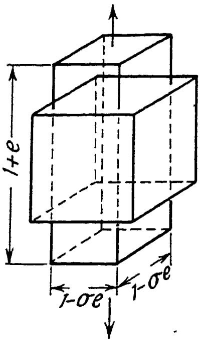


FIG. 12. LATERAL CONTRACTION RESULTING FROM ONE-DIMENSIONAL STRESS

Suppose a cube of unit side to be subjected to uniform tension over one face, Fig. 12. Each stretched edge extends to $1 + e$ and each transverse edge contracts to $1 - \sigma e$. The volume of the cube therefore changes from unity to $(1 + e)(1 - \sigma e)(1 - \sigma e)$.

The change of volume

$$\Delta = (1 - 2\sigma e + e + \sigma^2 e^2 - 2\sigma e^2 + \sigma^2 e^3) - 1$$

which, since the strain is small, is very nearly $e(1 - 2\sigma)$. If each face be subjected to the same pull the total increase in volume will be nearly three times as great or

$$3e(1 - 2\sigma)$$

If p is the applied force we have $e = P/E$ since we are dealing with unit area, and further, on defining Δ as p/K, the relation

$$K = E/3(1 - 2\sigma)$$

A body that possesses the same elastic properties in all directions is termed *isotropic*.

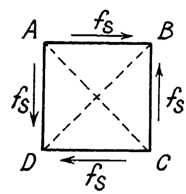


FIG. 13. UNIT CUBE OF MATERIAL UNDER EQUAL SHEARS ON TWO PAIRS OF OPPOSITE FACES

Relations Between the Elastic Constants E, N, K and σ. If shear forces f_s are applied to the opposite faces AB and CD of a unit cube as in Fig. 13, it is clear that for the cube to remain in equilibrium, a balancing couple is required of magnitude $f_s \times AD = f_s$ since AD is unity. This must be accomplished by equal and opposite shear stresses on faces AD and CB as shown in the figure.

For the portion ADC to be in equilibrium a pulling force $2(f_s/\sqrt{2}) = (\sqrt{2})f_s$ must act normal to the diagonal AC and for

the portion BCD to remain in equilibrium a force of amount $f = (\sqrt{2})f_s$ must act towards and normal to the diagonal plane BD. (Fig. 14 (a) and (b).)

The area of AC or AB is $\sqrt{2}$, as the cube is of unit side, and if f_t and f_c are the respective tensile and compressive stresses $f_t = (\sqrt{2})f_s/\sqrt{2} = f_s$. Similarly $f_c = f_s$.

It follows that shear stresses on planes at right angles to each other are equivalent to tensile and compressive stresses of

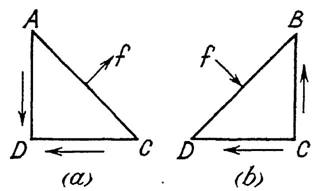


FIG. 14. NORMAL STRESSES ON DIAGONAL PLANES AS A RESULT OF THE STRESS SYSTEM OF FIG. 13

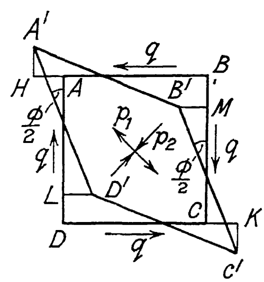


FIG. 15. DISTORTION OF UNIT CUBE BY SHEAR STRESSES

intensity equal to that of the shear stress acting on planes at right angles and inclined at 45° to the planes of shear stress.

Under the action of the shearing forces the cube $ABCD$ of Fig. 15 will be deformed to $A'B'C'D'$, and the shear strain will be given by the angular distortion ϕ.

On planes AC and BD only tensile and compressive forces act, as shown. It is convenient here to adopt a different notation. Let q represent the shear stress on the faces of the cube and p_1 and p_2 the tensile and compressive forces respectively on the diagonal planes. The diagonal AC undergoes a strain p_1/E and also a strain $\sigma p_2/\mathrm{E}$ so that the resulting strain is

$$\begin{aligned} p_1/\mathrm{E} + \sigma p_2/\mathrm{E} &= (1/\mathrm{E})(p_1 + \sigma p_2) \\ &= (q/\mathrm{E})(1 + \sigma) \end{aligned}$$

as $p_1 = p_2 = q$ in magnitude.

The diagonal BD undergoes a contraction of the same amount.

The strain consists of the horizontal displacements AH and CK on account of the horizontal shearing forces, and the vertical displacements HA' and KC' on account of the vertical shearing

forces. If the total shear is ϕ the angle of 90° between the faces of the cube is changed to $90 \pm \phi$ so that each face turns through $\pm \frac{1}{2}\phi$.

The extension along the diagonal AC

$$\begin{aligned}
&= AA' + CC' \\
&= (AH + CK)\,(1/\sqrt{2}) + (HA' + KC')\,(1/\sqrt{2}) \\
&= (A'D'/\sqrt{2})\,(\phi/2) + (A'B'/\sqrt{2})\,(\phi/2) \\
&= (AD + AB)\,(\phi/2\sqrt{2}) = 2 \times \phi/2\sqrt{2} = \phi/\sqrt{2}
\end{aligned}$$

as the sides of the cube are but slightly altered in length and $AD = AB = 1$.

The strain along the diagonal is therefore

$$(\phi/\sqrt{2}) \div \sqrt{2} = \phi/2.$$

But the strain along the diagonal is, as we have already seen,

$$(1 + \sigma)(q/\mathrm{E}),$$

hence $$(1 + \sigma)(q/\mathrm{E}) = \phi/2 = q/2\mathrm{N}$$

so $$\mathrm{N} = \mathrm{E}/2(1 + \sigma).$$

But $\mathrm{K} = \mathrm{E}/3(1 - 2\sigma)$; hence follow the relations

$$\sigma = \mathrm{E}/2\mathrm{N} - 1 = \tfrac{1}{2} - \mathrm{E}/6\mathrm{K}$$

and $$\mathrm{E} = 9\mathrm{NK}/(3\mathrm{K} + \mathrm{N})$$

connecting the elastic constants.

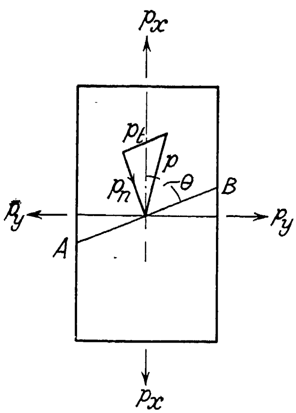


FIG. 16. STRESS COMPONENTS ON INCLINED SECTION OF A BAR SUBJECTED TO STRESSES AT RIGHT ANGLES

Principal Stresses and Planes of Stress. It can be shown that under any complex system of forces the stress at any point in a body can be resolved into a combination of three simple tensile or compressive stresses on three mutually perpendicular planes. Such stresses are called *principal stresses* and the planes are *principal planes* of stress at the point.

A simple case is that of tensile stresses p_x and p_y acting on a block in two directions at right angles, Fig. 16. On any plane AB inclined at θ to the vertical the resultant stress p can be resolved into a stress p_n normal to AB and a stress p_t tangential to AB.

In terms of the applied stresses

$$p_n = p_x \sin^2\theta + p_y \cos^2\theta$$
$$p_t = (p_x - p_y) \sin\theta \cos\theta.$$

and the resultant stress

$$p = \sqrt{(p_n^2 + p_t^2)} = \sqrt{(p_x^2 \sin^2\theta + p_y^2 \cos^2\theta)}$$

The inclination of this stress to the plane AB is given by

$$\tan\alpha = \frac{p_n}{p_t} = \frac{p_x \sin^2\theta + p_y \cos^2\theta}{(p_x - p_y)\sin\theta\cos\theta} = \frac{p_x \tan^2\theta + p_y}{(p_x - p_y)\tan\theta}.$$

If the stresses are unlike, taking p_x as tensile and p_y as compressive, the resulting normal and tangential stresses on the plane AB are

$$p_n = p_x \sin^2\theta - p_y \cos^2\theta$$
$$p_t = (p_x - p_y)\sin\theta\cos\theta.$$

For $p_x = p_y$ and $\theta = 45°$, we have $p_n = 0$ and $p_t = p_x$.

In this instance P_x and P_y are themselves principal stresses. But consider the wedge ABC of unit thickness Fig. 17, under known stresses p_x, p_y and q as indicated.

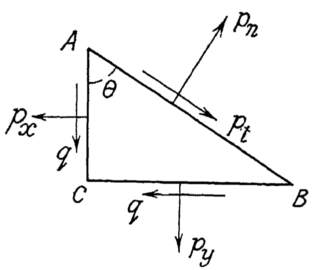


FIG. 17. WEDGE UNDER NORMAL AND SHEAR STRESSES

Resolving normally and tangentially to the plane AB inclined at θ to AC, we find for the normal stress

$$p_n = p_x \cos^2\theta + p_y \sin^2\theta + 2q \sin\theta\cos\theta$$

and for the tangential stress

$$p_t = (p_x - p_y)\sin\theta\cos\theta - q(\cos^2\theta - \sin^2\theta)$$
$$= \tfrac{1}{2}(p_x - p_y)\sin 2\theta - q\cos 2\theta$$

The value of θ that yields the greatest value of p_n is given by $\tan 2\theta = 2q/(p_x - p_y)$, and for the particular value of θ that satisfies this equation p_t is zero.

A twisting moment $T_e = \pi d^3/16$ will produce a pure shear stress and also a pure tensile stress of intensity p, and $T_e = M + \sqrt{(M^2 + T^2)}$ is termed the *equivalent twisting moment.*

It can be shown that the *equivalent bending moment* M_e that would produce the same maximum normal stress as M and T acting together is $M_e = \frac{1}{2}M + \frac{1}{2}\sqrt{(M^2 + T^2)}$.

All planes parallel to the principal planes will be subjected only to normal stresses so that our elementary prism *ABCD* of Fig. 18 will enclose smaller prisms such as *EFGH*, Fig. 19, on whose sides only the normal stresses p_1 and p_2 act. These have the values

$$p_1 = \tfrac{1}{2}p_x + \tfrac{1}{2}\sqrt{(p_x^2 + 4q^2)}$$

and $$p_2 = \tfrac{1}{2}p_x - \tfrac{1}{2}\sqrt{(p_x^2 + 4q^2)}$$

as shown previously.

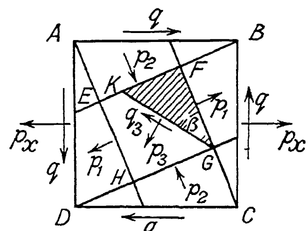


FIG. 19. $\beta = 45°$ IS THE PLANE OF MAXIMUM SHEAR STRESS

Consider now the equilibrium of the wedge of material *FGK* in the prism *EFGH*. On the face *GK* inclined at β to *GF* there exists a normal stress p_3, and a shear stress q_3. The latter is a maximum for $\beta = 45°$ and is then given by

$$q_3 = \tfrac{1}{2}(p_1 - p_2)$$

and in terms of p_x and q, the stresses with which we commenced,

$$q_3 = \tfrac{1}{2}\sqrt{(p_x^2 + 4q^2)}$$

Substituting for p_x and q their values in terms of the bending and twisting moments respectively

$$q_3 = (16/\pi d^3)\sqrt{(M^2 + T^2)}$$

A simple twisting moment $T_e = (\pi/16)d^3q_3$ would produce the same shear stress. Hence a twisting moment $T_e = \sqrt{(M^2 + T^2)}$ will produce the same maximum shear as M and T acting together.

If the ultimate resistance to rupture by shearing be less than half the resistance to rupture by direct tension or compression a material will fail by shearing when subjected to a direct tensile or compressive force.

The theory that mild steel shafts under combined bending and twisting fail through shear is due to Guest.

In cases of complex stress the plane of maximum stress is not necessarily the plane of maximum strain. This point is of importance in considerations of the criterion of strength; whether failure occurs by the stress exceeding a certain value or by the strain exceeding a certain value.

Suppose p_1 and p_2 to be two principal tensile stresses $p_1 > p_2$. The strain in the direction of p_1 is

$$p_1/\mathrm{E} - \sigma p_2/\mathrm{E} = (1/\mathrm{E})(p_1 - \sigma p_2)$$

The equivalent stress needed to produce the same strain is

$$p_e = p_1 - \sigma p_2.$$

Hence

$$\begin{aligned} p_e &= p_1 - \sigma p_2 \\ &= (p_x/2)[1 + \sqrt{(1 + 4q^2/p_x^2)}] - (\sigma p_x/2)[1 - \sqrt{(1 + 4q^2/p_x^2)}] \\ &= (p_x/2)[(1 - \sigma) + (1 + \sigma)\sqrt{(1 + 4q^2/p_x^2)}] \end{aligned}$$

At right angles to this there will be an equivalent stress

$$p_e^1 = (p_x/2)[(1 - \sigma) - (1 + \sigma)\sqrt{1 + (4q^2/p_x^2)}]$$

For steel, Poisson's Ratio lies between 0·25 and 0·3, hence

$$\begin{aligned} p_e &= (p_x/2)[(1 - 0{\cdot}25) + (1 + 0{\cdot}25)\sqrt{(1 + 4q^2/p_x^2)}] \\ &= \tfrac{3}{8}p_x + \tfrac{5}{8}\sqrt{(p_x^2 + 4q^2)} \text{ when } \sigma = 0{\cdot}25 \end{aligned}$$

and

$$p_e = 0{\cdot}35p_x + 0{\cdot}65\sqrt{(p_x^2 + 4q^2)} \text{ when } \sigma = 0{\cdot}3.$$

Graphic Representation of Stress. Referring to the equations for the normal and tangential stress components in terms of the principal stresses, namely,

$$p_n = \tfrac{1}{2}(p_1 + p_2) + \tfrac{1}{2}(p_1 - p_2)\cos 2\theta \qquad (1)$$

$$p_t = \tfrac{1}{2}(p_1 - p_2)\sin 2\theta \qquad (2)$$

these lead to a graphic representation known as *Mohr's Circle of Stress*, Fig. 20.

Let OA and OB represent the stresses p_1 and p_2 respectively. Then a circle on BA of radius $\frac{1}{2}(p_1 - p_2)$ will give the variation in the values of p_n and p_t for various values of θ.

For a given value of θ let CD be drawn making an angle 2θ with the direction of p_n and also the perpendicular DF on OA. From the figure,

$$OF = OC + CF = (p_1 + p_2)/2 + (p_1 - p_2) \cos 2\theta/2$$

$$= p_1 \cos^2 \theta + p_2 \sin^2 \theta$$

$$DF = CD \sin 2\theta = (p_1 - p_2) \sin 2\theta/2$$

so that OF represents the normal stress on the section and DF the tangential stress. The maximum normal stress p_1 is given

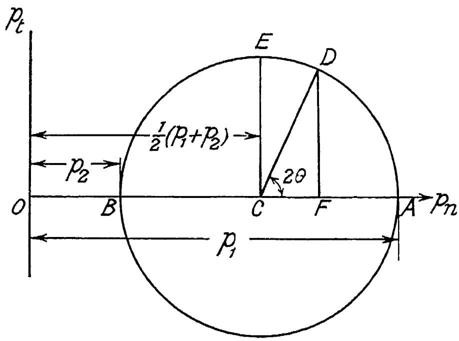


FIG. 20. MOHR'S CIRCLE OF STRESS

by OA and the maximum tangential stress $(p_1 - p_2)/2$ is given by CE and acts on the section corresponding to $\theta = 45°$.

Another useful graphical illustration is the ellipse of stress. Consider a unit cube of material to be subjected to normal stresses p_1 and p_2 as indicated in Fig. 21. Take OX in the direction of p_1 and OY as the direction of p_2. Consider the stresses on the plane AB. Let $OC = f$ be the resultant stress on this plane making an angle ϕ with the vertical. Let CD be drawn parallel to OX and CF parallel to OY. Then $OF = x = f \sin \phi$ and $OD = y = f \cos \phi$. The total force in the direction of OC is $f \,.\, AB$. Its vertical component $f \,.\, AB \,.\, \cos \phi$ balances the vertical force on AB that is $f \,.\, AB \,.\, \cos \phi = p_2 AB \cos \theta$. Its horizontal component $f \,.\, AB \,.\, \sin \phi$ balances the horizontal force on AB; that is

$$f \,.\, AB \,.\, \sin \phi = p_1 AB \sin \theta$$

Hence

$$(f^2 \sin^2 \phi)/p_1^2 + (f^2 \cos^2 \phi)/p_2^2 = \sin^2 \theta + \cos^2 \theta = 1$$

Substituting x and y for $f \sin \phi$ and $f \cos \phi$ we have

$$x^2/p_1^2 + y^2/p_2^2 = 1$$

the equation of an ellipse—the ellipse of stress. Its semi-major axis is p_1 and its semi-minor axis p_2 and these are the radii of the outer and inner circles respectively.

To find the stress on any plane such as AB, draw the normal ON. Draw the perpendicular on OY to intersect the ellipse in

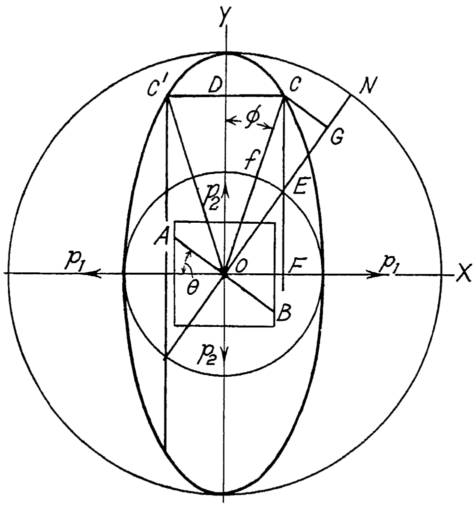


FIG. 21. ELLIPSE OF STRESS

the point C. Then OC gives the magnitude and direction of the resultant stress. A perpendicular CG on the normal will give the normal stress $p_n = OG$ and the tangential stress $p_t = CG$.

If p_1 were negative it would need to be measured in the opposite direction. The resultant stress would then be given by OC'.

Mohr's representation may be extended to three-dimensional systems of stress. Suppose a rectangular bar to be subjected to normal stresses p_1, p_2, p_3 over faces perpendicular to the x-, y-, z- axes respectively, $p_1 > p_2 > p_3$. Over a section through the z- axis the stresses p_n and p_t may be calculated by means of equations 1 and 2, page 18. The circle (1), Fig. 22, represents these stresses. The circle (2) represents the stresses over any section through the x-axis. Circle (3) represents the stresses

on any section through the y-axis. The three circles represent the stresses over three families of sections through the x-, y-, z- axes. For any section inclined to the x-, y- and z- axes the stress components are given by the co-ordinates of a point situated in the shaded portion of the figure. Mohr's Theory, which is an extension of the maximum-shear theory, is dealt

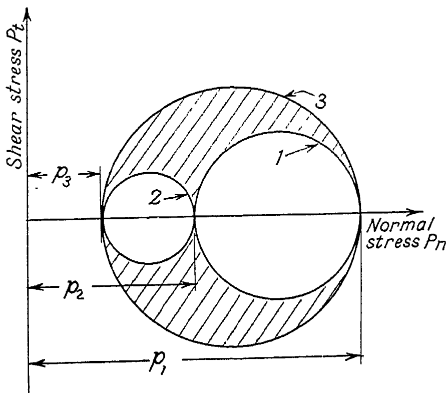


FIG. 22. MOHR'S REPRESENTATION EXTENDED TO SYSTEMS OF THREE DIMENSIONS

with in the works marked with an asterisk in the Bibliography, page 261.

Struts. Cases arise in which a member under load is brought to a state of unstable equilibrium and failure occurs by buckling.

The most important example is that of a strut which, loaded axially, will fail under a load lower than that needed to cause failure through direct stress. The length of the strut and the geometrical distribution of the material in the cross-section are important factors influencing the crippling load.

With struts which are long compared with their lateral dimensions, the collapsing load is given by Euler's formula

$$P = \pi^2 EI/L^2$$

where P = the collapsing load;
E = Young's modulus for the material;
I = the least moment of inertia of the cross-section:
L = the length of the strut supposed pin-jointed or hinged at the ends.

The safe working load is obtained by introducing a suitable factor of safety.

For a given strut the conditions obtaining at the ends greatly affect the collapsing load.

There are four standard cases, Fig. 23—

Case I. Ends pin-jointed or free to take up any angle of slope as in Fig. 23 (*a*). Collapsing load $P = \pi^2 EI/L^2$.

Case II. Both ends fixed in position and direction, Fig. 23 (*b*). Collapsing load $P = 4\pi^2 EI/L^2$.

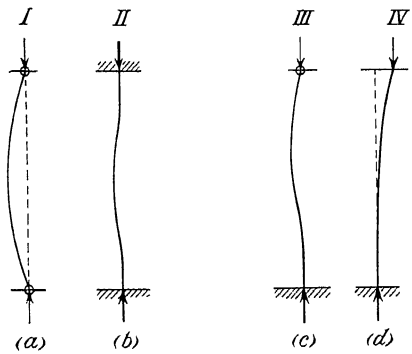


FIG. 23. END CONDITIONS FOR STRUTS—STANDARD CASES

Case III. One end rigidly fixed and the other hinged. Fig. 23 (*c*). Collapsing load $P = (81/4)(EI/L^2)$.

Case IV. One end fixed and the other free to move laterally and to take up any angular position, Fig. 23 (*d*). Collapsing load $P = \pi^2 EI/4L^2$.

The theoretical formulae are derived on the assumption that the struts are perfectly straight and homogeneous and that the load is applied in a perfectly axial manner. Actual struts seldom satisfy the theoretical conditions and various empirical formulae are employed to calculate the collapsing load. Rankine's formula is

$$P = \frac{f_c A}{1 + a(L/K)^2}$$

where

A = area of cross-section;

f_c = the intensity of the ultimate compressive stress, a

quantity difficult to determine experimentally and sometimes taken as the stress at the yield point in compression;

k = the least radius of gyration of the cross-section;

a = a constant for a strut with both ends free. Case I.

For Case II the constant becomes $a/4$
,, III ,, ,, $a/2$
,, IV ,, ,, $4a$

Values of f_c and a usually given are—

Material	f_c tons per in.^{2}	a
Hard steel . .	30	1/5 000
Mild steel . .	21	1/7 500
Wrought iron . .	16	1/9 000
Cast iron . .	36	1/1 600

Strain Energy. Suppose a structural member or a test piece to be loaded gradually by increasing the load uniformly from zero. If, when the extension amounts to s, Fig. 24, the load has

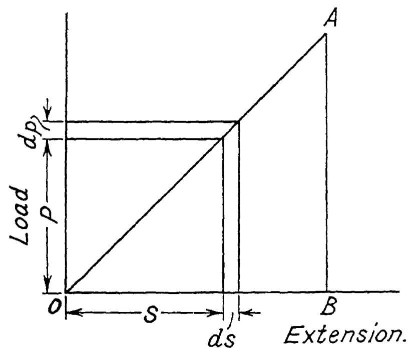


FIG. 24. AREA *AOB* REPRESENTS WORK DONE DURING ELASTIC STRAIN

a value P, then for an extension $s + ds$ the load will be $P + dP$. The average load over the increment of extension ds is $(P + dP/2)ds = Pds$, neglecting the small product $(dP \times ds)/2$. The total work done in straining the member under the assumed conditions is therefore represented by the area OAB and $= \int Pds$.

Now s = strain $\times$ length = PL/AE and therefore ds = (L/AE)dP. Hence the total work is

$$\mathrm{U} = \int \frac{\mathrm{PL}}{\mathrm{AE}} d\mathrm{P} = \frac{\mathrm{P}^2\mathrm{L}}{2\mathrm{AE}}$$

for a member of length L and cross-section A.

The result is obtained on the assumption that the load is applied gradually so that the whole work performed is stored as potential or strain energy in the member. Actually it is immaterial whether the load be applied gradually or not, provided the material is not overstrained in the process, since the excess energy over and above the amount $\frac{1}{2}\mathrm{P} \times s$ is transformed into heat during the damping of the vibrations set up.

The strain energy stored in a member due to bending can be shown to be

$$\mathrm{U} = \int \frac{\mathrm{M}^2}{2\mathrm{EI}} dx$$

the bending moment M being variable along the length x of the beam.

Similarly, the strain energy due to shear is

$$\mathrm{U} = \int \frac{k\mathrm{F}^2}{2\mathrm{AN}} dx$$

where A is the area of the cross-section, N the modulus of rigidity, and F the shear force.

The factor k is introduced because the shear stress is not uniformly distributed over the section, the distribution depending on the shape of the section and the loading.

For a circular shaft subjected to a torque T the strain energy is

$$\mathrm{U} = \int \frac{\mathrm{T}^2}{2\mathrm{NJ}} dx \quad \text{or} \quad \frac{\mathrm{T}^2\mathrm{L}}{2\mathrm{NJ}}$$

if T be constant throughout the length L.

The energy stored per cubic inch of material when stressed to the elastic limit is termed the *resilience*. For a tensile stress p_t, compressive stress p_c or shear stress p_s, the resiliences are $p_t^2/2\mathrm{E}$, $p_v^2/2\mathrm{E}$ and $p_s^2/2\mathrm{N}$ respectively.

If the unit volume of material be subjected to two principal stresses p_1 and p_2 the work done per unit volume may be found as follows—

The strain in the direction of p_1 is

$$e_1 = p_1/\mathrm{E} - \sigma p_2/\mathrm{E}$$

and in the direction of p_2 is

$$e_2 = p_2/\mathrm{E} - \sigma p_1/\mathrm{E}.$$

Hence under gradual application of the stress the work done is

$$\mathrm{W} = \tfrac{1}{2}p_1e_1 + \tfrac{1}{2}p_2e_2 = p_1^2/2\mathrm{E} + p_2^2/2\mathrm{E} - \sigma p_1p_2/\mathrm{E}$$

Similarly, if at a point three principal stresses p_1, p_2, p_3 act, the work done per unit volume of material in producing these stresses is

$$\mathrm{W} = (1/2\mathrm{E})(p_1^2 + p_2^2 + p_3^2) - (\sigma/\mathrm{E})(p_1p_2 + p_2p_3 + p_3p_1)$$

in which a tensile stress is regarded as positive and a compressive stress as negative.

Theories of Elastic Failure. The chief theories put forward to account for the elastic failure of materials from which the strength of a material under combined stress may be deduced from the results of simple tests in tension and compression are—

(*a*) *The Maximum Stress Theory*, sometimes called Rankine's Theory, which assumes that, in ductile materials, yielding starts in an element when the maximum tensile stress becomes equal to the yield point of the material in simple tension, or the maximum compressive stress becomes equal to the yield point of the material in simple compression. This theory is contradicted by many examples.

For instance, if the theory is always true the shearing elastic limit must be at least equal to the tensile elastic limit. But for nearly all metals the elastic limit in shear is much less than the elastic limit in tension.

(*b*) *The Maximum Strain Theory* or St. Venant's Theory, which assumes that the yielding of a ductile material starts when the maximum strain becomes equal to the strain at which yielding occurs in simple tension. Results show that this theory conflicts with practice in many cases.

(*c*) *The Maximum Shear Theory* or Guest's Theory. This assumes that elastic failure begins when the maximum shear stress becomes equal to the maximum shear stress found at the yield point in simple tension. Since the maximum shear stress is equal to half the difference between the maximum and minimum principal stresses the condition for yielding is that

$$\tfrac{1}{2}(p_1 - p_2) = \tfrac{1}{2}\text{ (yield point in tension).}$$

The maximum shear theory agrees better with experiment than either of the foregoing theories, and in machine design is often used in the case of ductile materials. The precise determination of the yield point in shear is not easy.

(*d*) *Haigh's Strain Energy Theory.* This states that inelastic action at any point in a body due to any combination of stresses begins only when the energy per unit volume absorbed at the point is equal to the energy absorbed per unit volume by a bar when stressed to the elastic limit in simple tension.

The condition for yielding is when

$$\frac{1}{2\mathrm{E}}(p_1^2 + p_2^2 + p_3^2) - \frac{\sigma}{\mathrm{E}}(p_1p_2 + p_2p_3 + p_3p_1)$$

$$= \frac{(\text{stress at yield point})^2}{2\mathrm{E}}$$

The above theories are compared graphically in Fig. 25 for the case of two principal stresses. The lines in the diagram represent the values of p_1 and p_2 at which yielding starts according to the several theories. The maximum stress theory is represented by the square *ABCD*. The lengths *OA*, *OB* represent the yield points in simple tension in the x and y directions respectively. Points *C* and *D* correspond to compression.

Point *a* represents equal tensions in two perpendicular directions, each equal to the yield point in simple tension.

By the maximum stress theory there is no yielding for any point inside the square. The maximum strain theory is represented by the rhombus *efgh*. Since a tension in one direction reduces the strain in a perpendicular direction, two equal tensions according to this theory can have much higher values at yielding, represented by the point *e*, than with the maximum stress theory, point *a*. If the two principal stresses are equal and opposite in sign, yielding starts at lower values according to the maximum strain theory—points *f* and *h*—than with the maximum stress theory.

The hexagon *AaBCcD* represents the maximum shear theory. The results given by the maximum shear and maximum stress theories coincide when both principal stresses are equal, but there is considerable difference when the principal stresses are of opposite sign.

In the case of two dimensions we have by the strain energy equation $p_1^2 + p_2^2 - 2\sigma p_1 p_2 = (f_y)^2$ where f_y is the stress at the yield point.

This is the equation of an ellipse enclosing all points at which no yielding takes place according to the maximum strain energy theory. It is represented by the full line curve in Fig. 25. As we have seen, the figure enclosing all points at which no yielding takes place according to the maximum shear theory is a

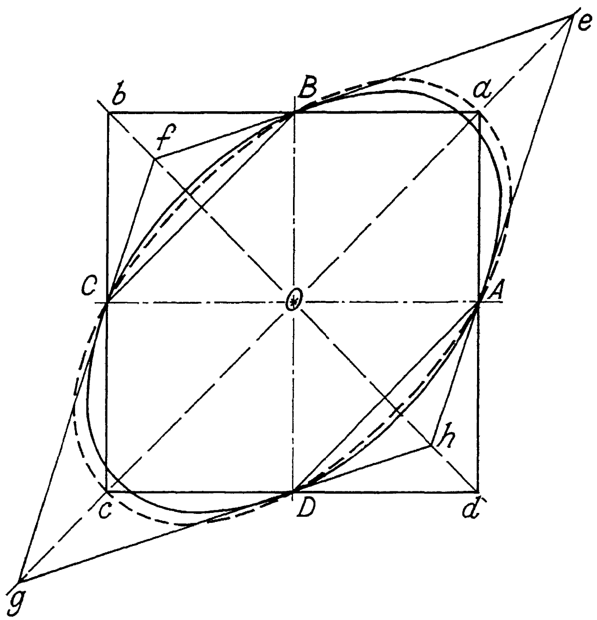


FIG. 25. GRAPHIC COMPARISON OF THE THEORIES OF ELASTIC FAILURE

hexagon. In a three dimensional stress system the corresponding surface is a hexagonal prism; this limiting surface consisting of six different planes in the stress space.

To avoid the discontinuities associated with the maximum shear theory Hencky and von Mises assumed the limiting surface to be capable of representation by an equation of the form

$$(p_1 - p_2)^2 + (p_2 - p_3)^2 + (p_3 - p_1)^2 = \text{constant}$$

the equation of a cylinder circumscribed about the hexagonal prism. It was later shown by Hencky that the expression on the left-hand side of this equation represents, except for a constant factor, the elastic energy stored in the material in shear.

If e_1, e_2, e_3 be taken as the principal strains, the total energy stored in unit volume of material is

$$(p_1e_1 + p_2e_2 + p_3e_3)/2$$

Substituting for the values of the strains in terms of the principal stresses by using the equations (page 12)

$$e_1 = [p_1 - \sigma(p_2 + p_3)]/\mathrm{E} \text{ etc.}$$

we obtain

$$(1/2\mathrm{E})[p_1^2 + p_2^2 + p_3^2 - 2\sigma(p_1p_2 + p_2p_3 + p_3p_1)]$$

The work absorbed in changing the volume, namely

$$\tfrac{1}{2}[(p_1 + p_2 + p_3)/3](e_1 + e_2 + e_3)$$

is equal to

$$[(1 - 2\sigma)/6\mathrm{E}](p_1 + p_2 + p_3)^2$$

when the foregoing values of e_1, e_2, e_3 are substituted.

Subtracting this work from the total energy stored, there remains for the energy (W) of distortion

$$\begin{aligned} \mathrm{W} &= [(1 + \sigma)/6\mathrm{E}][(p_1 - p_2)^2 + (p_2 - p_3)^2 + (p_3 - p_1)^2] \\ &= (1/12\mathrm{N})[(p_1 - p_2)^2 + (p_2 - p_3)^2 + (p_3 - p_1)^2] \end{aligned}$$

since $\mathrm{N} = \mathrm{E}/2(1 + \sigma)$.

For simple tension where two of the principal stresses are zero

$$\mathrm{W} = (1 + \sigma/3\mathrm{E})p_1^2 = p_1^2/6\mathrm{N}$$

The value of the constant in the equation of the surface is thus

$$12\mathrm{NW} = 12\mathrm{N}(p_1^2/6\mathrm{N}) = 2p_1^2 = 2f_y^2$$

where $f_y = p_1$, is the yield stress in simple tension.

In a two dimensional stress system we have

$$(p_1 - p_2)^2 + (p_2^2) + (-p_1)^2 = 2f_y^2$$

that is

$$p_1^2 + p_2^2 - p_1p_2 = f_y^2$$

the equation of an ellipse. This is shown dotted, circumscribing the hexagon, Fig. 25.

Example. To illustrate the foregoing theories consider a shaft of diameter d subjected to a twisting moment $\mathrm{T} = 25$ ton-inches and a bending moment $\mathrm{M} = 33$ ton-inches, the elastic limit of the material being 20 tons per in.^{2} in tension and in shear 0·5 of this value. Suppose the shaft to be strained to half

the elastic limit. The shear stress $q = 16\text{T}/\pi d^3$ and the tensile stress $p = 32\ \text{M}/\pi d^3$.

Maximum Stress Theory.

The maximum normal stress

$$= \tfrac{1}{2}p + \tfrac{1}{2}\sqrt{(p^2 + 4q^2)}$$

Hence $$\frac{20}{2} = \frac{1}{2}\cdot\frac{32\text{M}}{\pi d^3} + \frac{1}{2}\sqrt{\left[\left(\frac{32\text{M}}{\pi d^3}\right)^2 + \left(\frac{2 \times 16\text{T}}{\pi d^3}\right)^2\right]}$$

Substituting for M and T and solving for d, we find

$$d = 3{\cdot}355 \text{ in.}$$

Maximum Strain Theory, assuming $\sigma = 1/3$.

Maximum stress $= 0{\cdot}35p + 0{\cdot}65\sqrt{(p^2 + 4q^2)}$,

and gives $$d = 3{\cdot}395 \text{ in.}$$

Maximum Shear Theory.

In this case the allowable limit of stress is reached when the shear stress $\frac{1}{2}\sqrt{(p^2 + 4q^2)}$, attains the value $0{\cdot}5 \times 20/2 = 5$ tons per in.^{2}. The required value of $d = 3{\cdot}48$ in.

Strain Energy Theory.

The energy absorbed per in.^{3} of material is $w = p^2/2\text{E} + q^2/2\text{N}$. But $\text{N} = \frac{2}{5}\text{E}$ for steel, hence $w = p^2/2\text{E} + 5q^2/4\text{E}$.

With the same working stress and E = 13 400 tons per in.^{2} the permissible amount of energy that may be absorbed per unit volume of material is

$$w_{max} = \frac{1}{2}\frac{f^2}{\text{E}} = \frac{1}{2}\frac{(20/2)^2}{13\ 400} = 0{\cdot}00373 \text{ inch-tons}$$

Hence, after substitution,

$$4 \times 0{\cdot}00373 \times 13\ 400 = 2(32\text{M}/\pi d^3)^2 + 5(16\text{T}/\pi d^3)^2$$

and $$d = 3{\cdot}395 \text{ in.}$$

Using von Mises' Theory

$$p_1 = \frac{1}{2}\left(\frac{32\text{M}}{\pi d^3}\right) + \frac{1}{2}\sqrt{\left[\left(\frac{32\text{M}}{\pi d^3}\right)^2 + \left(\frac{2 \times 16\text{T}}{\pi d^3}\right)^2\right]}$$

$$p_2 = \frac{1}{2}\left(\frac{32\text{M}}{\pi d^3}\right) - \frac{1}{2}\sqrt{\left[\left(\frac{32\text{M}}{\pi d^3}\right)^2 + \left(\frac{2 \times 16\text{T}}{\pi d^3}\right)^2\right]}$$

Substitution in the equation

$$p_1{}^2 + p_2{}^2 - p_1p_2 = f_y{}^2$$

gives $$100(d^3)^2 = (32^2/\pi^2)(\mathrm{M}^2 + \tfrac{3}{4}\mathrm{T}^2)$$

whence $$d = 3{\cdot}43 \text{ in.}$$

The relative diameters given by the several theories vary with the conditions specified.

CHAPTER II

THE STRUCTURE OF METALS

THE view of the elastician that materials are isotropic, on which view the analysis of the preceding chapter is based, holds good only with materials such as glass, quickly cooled slags and varnish. These materials are structureless even when viewed under the highest resolving power of the microscope. With metals, however, it has been established beyond doubt that all metallic substances are aggregates of crystals. A crystal may be defined as a substance whose atoms or molecules are arranged in a regular order, following generally some simple geometrical configuration. The regular outward form which we term a crystal is merely a necessary consequence of this internal symmetry. A crystal, therefore, has properties which are not uniform but which vary in different directions. When a member breaks with a "snap" fracture due to shock or fatigue, the crystalline nature is at once apparent and has led to the statement that "because the metal crystallized" it became brittle and broke off "short." This is quite a wrong impression. The material was crystalline from the very commencement of solidification and the crystalline appearance is not the cause, but the outcome, of the mechanism of fracture.

It is agreed that the elastic constants vary in different samples of nominally the same substance and the assumption that a metal or alloy is homogeneous or isotropic is inaccurate. Hence the fundamental assumption of the theory of elasticity is only a rough approximation to the truth. Nevertheless, in any metal or alloy, the crystal grains are generally so very small and their orientation of position, or *lie*, so much at random, that the material may be considered to be statistically homogeneous. This is especially so when we recollect that the sizes of members used in practice are invariably colossal when compared with the dimensions of the crystals of the material. Consequently, for the general purposes of strength of materials the assumption mentioned before may be admitted, and ordinarily metallic bodies may be considered to be continuous. As soon as precision is attempted the assumption fails completely and in any discussion of phenomena such as elastic failure, strain hardening,

hysteresis, yield, recovery, etc., the fine structure of metals and alloys cannot be left out of consideration. Matter exists in the three states, gaseous, liquid, and solid. The mechanism of transition of matter from one to another of these states is controlled by physical and not by chemical laws. The freezing of steel in an ingot is a process similar precisely to the freezing of water in a pond—both are examples of the change of a physical state from liquid to solid. So also, the condensation of steam in a vessel is the same phenomenon as the condensation of zinc vapour during the manufacture of zinc by the retort process. At the moment of solidification the atoms of the metal arrange themselves in a regular order. During this intense activity of the atoms the rate of cooling is arrested and an amount of energy, as heat, is evolved. This is the well-known *latent heat* of freezing. In the case of most pure commercial metals the atoms arrange themselves in the form of cubes, but zinc, magnesium and antimony crystallize in hexagonal form.

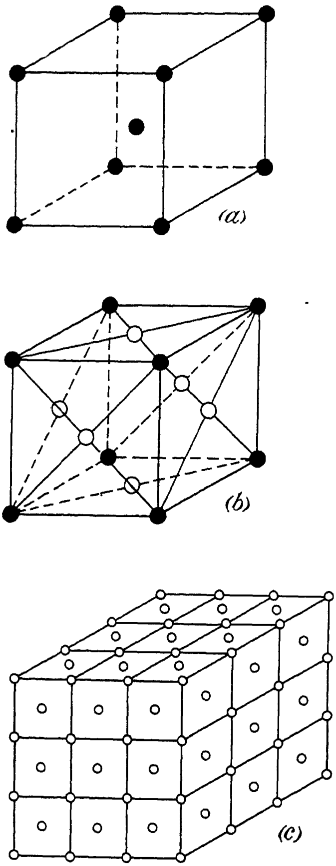


FIG. 26. GRAPHIC REPRESENTATION OF ATOMIC STRUCTURE

The atoms of a metal which crystallizes in the form of cubes, i.e. in the cubic system, may arrange themselves in two ways—(1) They may occupy the corners of a cube with one atom occupying the centre of each cube, Fig. 26 (*a*). This arrangement is called the *body-centred cube* arrangement and it is seen that each crystal unit is composed of nine atoms, kept in their relative positions by forces of attraction and repulsion existing between them. (2) They may occupy the corners of a cube with

one atom occupying the centre of each face, Fig. 26 (*b*). In this case each crystal unit will be composed of fourteen atoms, the arrangement being called the *face-centred-cube* arrangement.

The circles in the figure must not be regarded as representing the actual atoms but merely the spaces within which the atoms vibrate.

Since the atoms are not, as a rule, stationary, but are vibrating, the amplitude of the vibrations increases with the temperature up to a certain point where the vibration of the atoms will overcome the forces keeping them in their respective positions in the cube. At this temperature the regular arrangement is lost and the metal liquefies. Some metals are capable of crystallizing in more than one arrangement and are then said to possess different *allotropic* modifications. Each modification will possess different properties and will be stable at different limits of temperature. Thus, pure iron below 900° C. crystallizes as the body-centred-cube whilst above 900° C. it crystallizes as the face-centred-cube arrangement. Therefore, on cooling, pure iron changes its crystalline form at 900° C. Coincident with these changes is, amongst other things, the loss of the power to dissolve carbon. It is upon this allotropic transformation, which occurs at 900° C., that the whole of the heat treatment of steel depends. There is necessarily a space between the atoms which build up any solid material, so our present idea of a solid is one in which the constituents are built up as a lattice and we thus get the conception of a *space lattice*.

At the moment of solidification the atoms of the substance which have now almost freedom of motion but are under more restraint than when in the gaseous state, are subjected to the crystallizing force which arranges them on a regular space lattice. Some idea of the immensity of this force is shown by the splitting of rocks due to the freezing of water in pores and capillaries. Solidification starts by the simultaneous appearance of nuclei in various parts of the molten metal. Once the nuclei have made their appearance further solidification takes place by the building up of the atoms of the solid in a regular manner around each nucleus. From the nucleus, branches in three directions in space appear. These will have further branches and so on, all in a regular manner, until the solidifying mass of regular space lattice has grown to meet the solid grown in the same manner from another nucleus.

The atoms of the crystallizing material governed by any one

nucleus, will form crystal units which will arrange themselves regularly in one direction, which is generally referred to as the *orientation*. Similarly with the matter crystallizing from other nuclei, the crystal units will arrange themselves regularly in a different direction and this crystallized mass is said to possess a different orientation.

At this stage the solidifying mass is composed of a number of colonies each of which is formed on its nucleus and has been built up of a number of crystal units. The outward shape of the colony is symmetrical, and if the liquid remaining is poured or drains away the solidified mass has the regular shape usually associated with crystals. The perfectly grown crystal is known as an *idiomorphic* crystal. Ordinarily, however, the regular outward shape of a crystal is not seen because the colony, or what is a better term, the *crystal grain*, interferes and is interfered with by its neighbours and the outward form bears no resemblance to a crystal. It is, nevertheless, a crystal in the true sense as it is built up of material in a regular manner and its atoms are positioned on a regular space lattice. This crystal is termed an *allotriomorphic* crystal. The number of nuclei that will appear and hence the number of crystal grains in any particular volume will depend, among other things, on the rate of cooling in the immediate neighbourhood. The greater the rate of cooling the greater the number of crystal grains and generally the tougher the material. The atoms present in the last traces of liquid to solidify will be subjected to the crystallizing or arranging force of contiguous crystal grains and will not be able to attach themselves, as the pull exerted by one grain will balance that of another. The result is that the last remaining atoms will not form part of a crystal but will remain on their irregular or non-crystalline lattice. This film has been given the name *amorphous cement* but a more descriptive term is *irregular lattice*. This boundary between the crystal grains plays an important part. It is harder than the crystal proper, and its atoms not being regularly arranged, there is no crystalline material present and hence no favourable plane of cleavage. Consequently, in metals, fracture should and does ordinarily take place across the cleavage planes of the crystals and not between the crystals, i.e. in the boundaries.

If fracture should take place under normal conditions along the boundaries of the crystal grains a weak or brittle constituent in the boundary material is implied.

crystallize out simultaneously giving a characteristic laminated pattern. The structure is also grain-like since the eutectic appears as grains or colonies and the laminations of the two constituent metals are parallel in any grain, but the parallel laminations change their direction on going from one grain to another, each eutectic grain having originated in a nucleus and being controlled by it.

If the amount of this second metal is very small, the main or primary crystal will attract its own constituent from the eutectic, and as a result the alloy may consist of crystals of the metal in excess surrounded by an envelope of the second metal. If such a film were composed of a much weaker metal than the parent metal, then the strength of the alloy would be affected adversely and fracture would take place, not, as usual, through the cleavage planes of the crystal but between the crystals. For example, it is known that 0·01 per cent of bismuth will utterly destroy the resistance of copper to deformation. Again, brittleness in mild steel is associated with a film of hard carbide around the iron crystals. A weak film is not always a disadvantage, as a small amount of lead, which will not dissolve in copper or brass, but will separate between the crystal grains, will facilitate rapid machining.

On the other hand, a second metal may act as a stiffener for the rest of the alloy. A slight excess of zinc over that required to form what is known as the α – *solid solution* of zinc in copper causes the appearance of a second solution—the β – *solid solution* —containing more zinc, which being harder acts as a stiffening agent. A large amount of the β – solid makes the brass too hard for cold forging and hot forging must be resorted to.

(3) The two metals, on solidification, may combine in some simple atomic proportion to form a compound which, under the circumstances of the existence of the alloy, is indestructible and plays the part of an element or of a pure substance.

The freezing of this alloy will then take place exactly as in (1) or (2), depending on whether the compound is soluble or insoluble in the metal in excess of that required for the formation of the compound. Intermetallic compounds are generally brittle, and their formation must be controlled in all engineering materials.

It must be pointed out that the foregoing generalizations apply with equal force to alloys consisting of more than two metals, and further, that more than one solid solution and/or

more than one intermetallic compound may be formed in an alloy.

The temperatures at which an alloy undergoes changes such as solidification are generally accompanied by evolutions of heat on cooling and absorptions of heat on heating. If these temperatures are plotted on a chart having the horizontal axis for the composition, and the ordinate as temperature, the resulting chart is called the *equilibrium diagram.* In the case of two metals the equilibrium diagram is a plane figure, but in the case of three the diagram becomes three-dimensional.

The physical changes which a metal or alloy can undergo are not necessarily ended on solidification. Mention has already been made of the allotropic modifications of pure metals, and that on changing from one form to another the properties of the material may, and generally do, change. Another example is the fact that an alloy which at a high temperature forms a solid solution, at some lower temperature may break down into its constituents by a mechanism exactly parallel to the breakdown of a liquid solution into constituents which are not soluble in each other. So complete is the analogy that the eutectic formed from the breakdown of a liquid solution finds its counterpart in the *eutectoid* formed from the breakdown of a solid solution. These changes involved thermal changes which, with modern apparatus, are easily determined. It is upon a knowledge of the changes taking place in the solid alloy that the whole of the heat treatment of alloys depends.

It has already been stated that iron changes its crystal character at 900° C.; above this temperature, iron is in the γ-condition (or face-centred cubic lattice). Below this modification it is called *α-iron* (and is body-centred cubic). Iron when in the γ-condition can dissolve carbon. Consequently, just as magnetic properties are acquired on cooling through 900° C., so is the power of holding carbon in solid solution lost; the carbon, however, is not precipitated as such but as iron carbide, Fe_3C.

It must be remembered, then, that just as the addition of a second metal will lower the temperature at which freezing will start, so will carbon or iron carbide decrease the temperature at which the $\gamma \rightarrow \alpha$ change will take place. However vigorously an iron or steel is quenched the $\gamma \rightarrow \alpha$ change can not be wholly stopped and the speed at which steel cools through this critical range of temperature will affect the structure.

If a steel be heated to about 20° to 30° C. above this critical

temperature for about one hour per inch thickness of section the carbide of iron will go into solid solution due to the $\alpha \rightarrow \gamma$ change taking place.

If the steel be allowed to cool down with the furnace in its own time, the treatment is called *annealing*. If the bar is withdrawn and allowed to cool in air, it is said to be *normalized*, but if the cooling is quickened by plunging the bar straight into a liquid, the bar is said to be *hardened by quenching*—the hardness increasing with the rapidity with which the heat is withdrawn—quenching in cold water giving a much harder product than quenching in oil.

The mechanical properties depend on the type of cooling adopted: annealing gives relatively low tensile figures with relatively high elongations and comparatively low shock-resisting properties, whereas quenching in water gives high tenacity, low elongation, and low resistance to impact if the proportion of carbon be more than 0·25 per cent. Normalizing is roughly intermediate, with good shock-resisting properties.

With relatively slow cooling the nuclei of the α-iron crystals will form in that part of the structure of the γ-solid solution where the atoms are not already on a regular space lattice, i.e. in the grain boundaries, and a structure similar to the γ-iron, but not necessarily of the same grain size, will result if no carbon is present; whereas, if carbon be present the carbide of iron and part of the iron will form a eutectoid, the relative amounts of α-iron and eutectoid depending on the amount of carbon originally in the steel. With a rapid quenching, time is denied for the growth of α-crystals from nuclei and the α-crystals will be precipitated in the weakest part of the iron, i.e. in the cleavage planes. The face-centred crystal unit cubes of γ-iron arrange themselves as octahedra whose section on a cleavage plane is an equilateral triangle. The α-iron is then precipitated in these cleavage planes and can be seen under the microscope with suitable magnification, like crystal plates arranged at 60° to one another. This structure is typical and denotes a very brittle material. It is not confined to iron-carbon alloys (steels), but is found in any alloy system which crystallizes in a similar way, such as aluminium-copper-alloys (Al-Bronze) containing more than 10 per cent of aluminium.

The preparation and examination of metals and alloys under

the microscope requires a technique which is attained only by practice. A suitable specimen is carefully cut from the bulk and ground down to a mirror-like finish by means of emery papers of increasing degrees of fineness, the last traces of fine scratches being removed by polishing the specimen on a rapidly rotating cloth-covered disc fed with some oxide abrasive, generally diamantine, which is calcined alumina Al_2O_3. This fine polishing causes a very thin surface layer of "flowed" metal which has, during its slight mobility, smeared the surface and filled the little valleys of the scratches.

The structure of the metal is then revealed by an etching reagent which un-builds the structure downwards by dissolving away, first of all, the smeared surface layer. Dilute alcoholic solutions of mineral acids, such as nitric acid, are suitable reagents. The etching is stopped when desired and the structure examined under a microscope by normal illumination.

We have seen now that in any metal the atoms are built up in a regular manner having a geometrical configuration, and we can visualize that certain planes will contain more atoms than others and will accordingly resist stresses better so that cleavage and fracture will take place on certain planes more readily than on others. It will be realized that the lattice of any metal can be distorted, but only within certain limits, and that once that limit is exceeded the lattice breaks down.

On the basis that the atoms of any two metals are different, it follows that, when any atoms of an added metal take the place of atoms on a regular lattice there is bound to be distortion and consequent hardening. Such is the case when zinc is added to copper. Copper crystallizes in the face-centred cubic arrangement. A certain number of zinc atoms can enter the lattice, and although distorting the lattice and making the material harder it is not until about 30 per cent of zinc is added that the lattice is distorted so much that it breaks down.

Brass containing up to 30 per cent zinc is a solid solution termed α, and the space lattice pattern of the crystal is still the face-centred cubic. This brass can be cold forged. When more than 30 per cent zinc is added the number of zinc atoms so distorts the lattice that it breaks down and a new lattice, the body centred cubic, is formed, which is not so ductile. This second space lattice is another solid solution called β.

Ordinary yellow brass or Muntz Metal contains 40 per cent zinc and consists of crystals of the α-solid solution surrounded

by the crystals of the β-solid solution. This amount of β-solid solution is sufficient to stiffen up the α-solid solution so that it can be forged only at high temperatures, the alloy being too brittle to be forged at ordinary temperatures.

The space lattice of a metal or alloy can also be deformed by mechanical means. When the lattice is only slightly distorted, on removing the stress the lattice reverts to its normal configuration. If, however, the limit of distortion is exceeded, the lattice is permanently deformed and the elastic limit has been passed and a permanent set appears. Above the elastic limit, i.e. in the plastic region, the crystals accommodate themselves by a process whereby the planes of atoms slip over one another. This can be seen on a previously polished and etched surface of a metal or ductile alloy by the appearance of parallel, more or less straight, lines known as *slip bands* which, in reality, are little steps on the surface which have been produced by minute slips occurring on some of the crystal planes. By this means the crystal grains are elongated, becoming fibre-like, so that when a stress is imposed gradually and continuously up to breaking point, the resulting fracture is fibrous, breaking on a cleavage plane of the long fibre.

If, however, a stress greater than the breaking load is placed suddenly on the specimen the crystal grain has no time to elongate, and fracture occurs across the cleavage plane of the non-elongated crystal grain and appears coarsely crystalline.

In this short summary dealing with the structure of metals sufficient has been said to enable the testing engineer to appreciate the importance of the inner structure of the materials with which he has to deal. For a more detailed account reference should be made to works on metallography.

CHAPTER III

UNIVERSAL TESTING MACHINES

Principles Underlying the Construction of Testing Machines. The essential parts of a machine for making tension and compression tests comprise a straining gear for applying the load to the test piece and apparatus for measuring the load, so arranged that the accuracy of the measuring device is unaffected by the distortion of the specimen.

The straining mechanism may consist of a hydraulic ram on which fluid pressure is exerted by means of an accumulator or by a pump; or the straining action may be accomplished by screw gearing, operated by hand or power.

Machines may be of the horizontal or vertical type, the former offering some advantage for heavy work although occupying more floor space.

For measuring the load or force on the specimen several methods are available. The chief of these are—

(1) An application of the principle of the steelyard employing a lever, or system of levers, provided with a movable counterpoise. Such machines are termed *single-lever* or *multiple-lever* machines, as the case may be.

(2) The use of a weighted pendulum, the load being determined by observing the angle through which the pendulum is deflected.

(3) The simple method of measuring the pressure of the fluid in the straining cylinder.

(4) Balancing the load by fluid pressure acting on a diaphragm.

While methods (3) and (4) largely obviate inertia stresses in the test piece they are not regarded with favour, and by responsible authorities machines embodying the weighbeam principle are preferred.

Considerations of space prevent the description of the many types that have been introduced and attention will therefore be confined to a few modern machines used for commercial testing and research.

Denison 15-ton Testing Machine. A typical single-lever

machine of moderate capacity is the 15-ton machine made by Messrs. Samuel Denison & Son Ltd., Leeds.

The principle of the machine is shown in Fig. 27. A vertical frame F supports the horizontal steelyard or beam L, at the knife-edge B, the angular motion of the beam being limited by stops SS. At C is a second knife-edge carrying the link D, to the lower end of which the specimen is attached. The other end of the specimen is gripped in the cross-head H connected by tie rods to the ram R working in the straining cylinder A. The downward movement of the ram under pressure loads the test piece. To balance the load, the poise w is moved along the steelyard until equilibrium is attained.

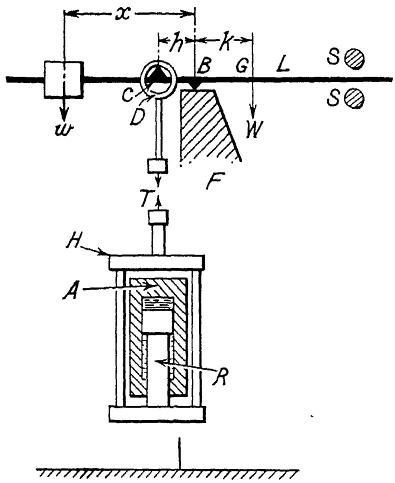


FIG. 27. PRINCIPLE OF SINGLE-LEVER TESTING MACHINE

If W is the weight of the steelyard acting at its centre of gravity G, at a distance k from the support, w the weight of the poise and T the pull on the test piece acting at a distance h from the support; then, for equilibrium,

$$\mathrm{T}h + wx - \mathrm{W}k = 0$$

The zero position, x_0 of the poise weight is obtained by placing $\mathrm{T} = 0$ in the above equation, whence

$$x_0 = \mathrm{W}k/w$$

The actual machine is illustrated in Fig. 28. For tensile tests the machine is equipped with dies for holding unprepared rounds, flats, and squares. The grips will accommodate flat specimens up to $\frac{3}{8}$ in. thick, and round and square specimens from $\frac{1}{4}$ in. to $1\frac{1}{4}$ in. diameter or side.

The maximum and minimum distances between the wedge boxes P are 18 in. and zero respectively.

Compression tests can be made on specimens up to 8 in. in length and the compression plates A are 7 in. diameter. The

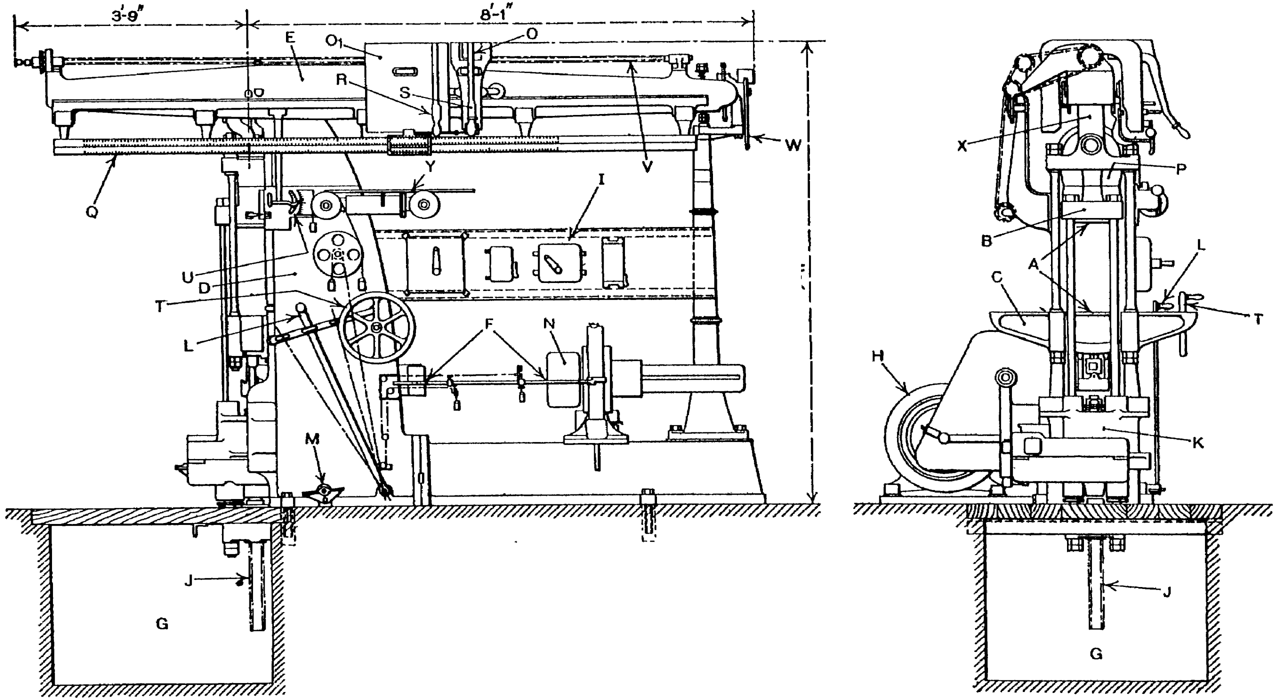


Fig. 28. Denison 15-ton Vertical Single-lever Testing Machine
(Machinery)

upper compression plate is attached to the underside of the straining head *B* by a simple bayonet fastening and the lower plate is located on the beam *C* by a spigot. The faces of the compression plates are scored with concentric rings to facilitate setting the specimen.

Transverse tests can be made on spans from 6 in. to 36 in. by 6 in. increments. Tests in double shear are performed by using the upper compression plate to apply the load to the middle member of a shear dog. The dog is provided with holes for testing specimens $\frac{1}{4}$ in., $\frac{3}{8}$ in., and $\frac{1}{2}$ in. diameter.

Bending tests on bars to be bent through 180° are carried out by the use of a special knee and block.

The controls are conveniently placed on the front of the main column *D*. Torsion tests are made at *F*, the maximum and minimum observed lengths of the torsion specimens being 20 and 3 in. respectively.

The straining gear is on the left of the main standard. No special foundations are required with the exception of a small pit *G* to take the straining crosshead.

The drive is by electric motor *H* through a roller chain and the motor is provided with a speed control panel *I*, the range of speed being from 350 to 1 400 r.p.m.

The straining gear receives its motion from one of the two chain drives from the motor to the lay-shaft and thence through a worm gear to the straining screw *J*. The two chain drives from the motor in conjunction with friction clutches controlled by the lever *L* provide two speeds, the upper being used for setting the head and the lower for applying the load.

The setting of a dog clutch controlled by a pedal *M* determines whether the lay-shaft drives the torsion straining head or the tension straining gear. For torsion at *F*, the drive passes through a compound worm box to the torque sleeve *N*, with which is incorporated a floating transmitter to allow for variation in the length of the torsion specimen when under test.

The weighing gear for use in tests other than torsion comprises a steelyard with a travelling poise weight OO_1. The knife-edges are of a patent dove-tailed inclined type, and as they are jig-produced they are interchangeable. The poise OO_1 is in two parts, a traveller and a follower, the former being $\frac{2}{15}$ of the total weight. With the traveller and follower disconnected the capacity of the machine is reduced to two tons. A hand lever *R* and catch on the follower enables the two to be readily

The Buckton 100-ton Horizontal Testing Machine is shown diagrammatically in Fig. 29, from which the principle of operation will be readily understood. In the straining cylinder C works a single-acting ram carrying the slide S. The slide is guided by a groove in the bed. The crosshead H_1 carried by the slide can be locked in any one of a number of positions by means of keys.

Tension specimens are held between the crossheads H_1 and H_2, the latter being attached to stout steel rods D running the

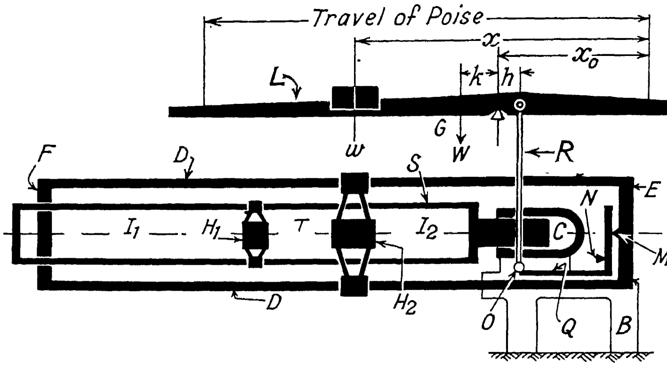


FIG. 29. BUCKTON 100-TON HORIZONTAL TESTING MACHINE

whole length of the machine. The rods are connected to the crossheads E and F, one at each end of the machine, the whole being termed the *measuring frame.*

The action of the machine will be understood by imagining a specimen inserted between H_1 and H_2 and pressure applied to the straining cylinder. As the ram moves outwards carrying with it the slide and its attached crosshead, the pull on the specimen is transmitted through the head H_2 and the rods D to the knife-edge M in the head E. The motion of E is constrained by the bell-crank lever Q and a system of knife-edges. The pull on the specimen is ultimately transmitted through the rod R to the steelyard L. By adjusting the poise to balance, the load on the test piece can be ascertained.

Compression and bending tests can be made by placing the specimen in either of the positions I_1, I_2. Bending tests are generally made by using a substantial cross beam as a support. This is attached to the head F and will accommodate specimens up to 10 ft. in length.

The friction between the sliding frame and the bed is of little import as the force to overcome frictional effects here is not transmitted to the measuring gear. Movements of the measuring frame, though of course very small, should not be restricted by friction, and to secure this condition as far as possible the frame is supported on a system of knife-edges and rollers.

In testing long members in compression, any side thrust caused by eccentricity of loading on the machine, or bending of the specimen, is transmitted by the slide to the bed without inconvenience. Some extra provision, however, is needed in the case of the measuring frame and this is secured by means of side stays fitted with rollers.

Pressure is applied to the straining cylinder by a hydraulic accumulator operating at 2 000 lb. per in.^{2} The return stroke of the ram was brought about in the earlier machines by the action of a heavy counterweight working in a pit under the bed, but this is now replaced by a hydraulic return cylinder.

The outer end of the ram at its point of attachment to the crosshead is threaded and carries a large circular nut. With this device it is possible to lock the specimen and maintain on it a constant load for as long as desired. Any trouble that might be occasioned by release of part of the load through fluid leakage is thus avoided.

The knife-edges *M* and *N* are carried by the bell-crank lever *Q*, their seats being formed by extensions on the head *E* and the cylinder *C* respectively. When the machine is operated at its full capacity the load on the knife-edges is 5 tons per in. run.

The joint *O* and the knife-edge *N* are on the same level and 32 in. apart. The vertical distance between *M* and *N* is 4 in., thus giving a leverage of 8 to 1.

A duplicate pair of knife-edges, not shown in the diagram, is provided at a distance of 8 in. below the level of the knife-edge *M*. This permits the leverage to be altered from 8 to 1 to 4 to 1 with a corresponding alteration of the scale of the weighbeam from 100 tons to 50 tons.

The poise w weighs $\frac{1}{2}$ ton and on the 100 ton scale the poise travels 2 in. per ton of load. With the 4 to 1 leverage the poise travels twice this amount or 4 in. per ton of load. On this scale it is possible to read to 0·001 ton, roughly $2\frac{1}{4}$ lb.

It is of interest to consider in some detail the forces acting on the steelyard. Referring to Fig. 29, if T_0 is the pull in the rod *R* due to the weight of the parts, W the weight of the beam

acting at its centre of gravity G, and w the weight of the poise acting at a distance x_0 from the fulcrum, then for equilibrium:

$$Wk = T_0h + wx_0$$

For a load P on the specimen producing a pull T in the rod R, we have, with the poise on the other side of the fulcrum and distant x from the zero of the scale,

$$\begin{aligned} Wk &= (T + T_0)h + w(x_0 - x) \\ &= Th + T_0h + wx_0 - wx \end{aligned}$$

and since

$$\begin{aligned} T_0h &= Wk - wx_0 \\ Wk &= Th + Wk - wx_0 + wx_0 - wx \end{aligned}$$

so that

$$Th = wx.$$

The distance h is 8 in. so that if the machine is used on the 100 ton range with the 8 : 1 leverage and if there is a pull of P tons on the specimen, $T = P/8$.

Hence $(P/8) \times 8 = wx = \frac{1}{2}x$, giving $P = x/2$ tons.

If P is 1 ton, $x = 2$ in., or the scale is 2 in. per ton of load, and 200 in. of travel are needed for the full 100 tons.

With the poise at the extreme end of its travel

$$\begin{aligned} 8\,(T + T_0) &= Wk + w(200 - x_0) \\ &= Wk + 134w \end{aligned}$$

as x_0 is about 66 in., T is then 100/8 and we have

$$8\,T_0 + \frac{100 \times 8}{8} = Wk + w(200 - x_0)$$

that is

$$\begin{aligned} 100 &= Wk + w(200 - x_0) - 8T_0 \\ &= Wk + 134\,w - 8T_0 \end{aligned}$$

Neglecting then the weight of the parts, we see that the force at the specimen when the poise is at the end of its travel is $134w$, and since w is $\frac{1}{2}$ ton the poise in this position balances about $\frac{3}{4}$ of the total load, the remainder being balanced by the weight of the beam itself.

Some interesting data relating to inertia effects in a machine of this type were given some years ago by the late Professor A. C. Elliott of University College, Cardiff.

The weight of the beam was about 1·25 tons and its moment of inertia about the fulcrum approximately 49 ton.-ft.^{2} units. The moment of inertia of the poise when at the end of its travel

about the fulcrum was 63 ton-ft.^{2} units. The total moment of inertia I of the beam and poise was thus 112 ton-ft.^{2}

Now, assuming the test piece to be stretching under an acceleration α, the angular acceleration of the steelyard will be $8\alpha/h$ and the torque about the fulcrum $(8\alpha/h) \times \mathrm{I}$.

The corresponding force at the specimen would be

$$\frac{8\mathrm{I}\alpha}{h} \times \frac{8}{h} = 112 \times \frac{8^2}{(8/12)^2} = 16\,100\alpha,$$

the reduced mass of the steelyard and poise being represented by 16 100 tons at the specimen.

The periodic time of vibration of the beam is given by

$$\mathrm{T} = 2\pi\sqrt{\frac{\text{angular displacement}}{\text{angular acceleration}}}$$

and this is the same as the period of oscillation of the specimen about its mean length, namely

$$\mathrm{T} = 2\pi\sqrt{\frac{\text{displacement}}{\dfrac{\text{force on specimen}}{\text{reduced mass}}}}$$

If L is the length of the specimen, A its cross-sectional area, E the modulus of elasticity, S the reduced mass, e the displacement, and $g = 32{\cdot}2 \times 12$ in the present instance,

$$\begin{aligned}\text{Force on specimen} &= \text{stress} \times \text{sectional area}\\ &= \mathrm{E} \times \text{strain} \times \text{sectional area}\\ &= \mathrm{E} \times (e/\mathrm{L}) \times \mathrm{A}\end{aligned}$$

and the time of vibration

$$\mathrm{T} = 2\pi\sqrt{\frac{\mathrm{LS}}{g\mathrm{EA}}}$$

For a wrought iron bar $1\frac{1}{2}$ in. diameter and 36 in. long, for which E was 12 000 tons per in.^{2}, and under a load of 20 tons for which the reduced mass of the beam was about 7 300 tons, the calculated time of oscillation was 1·12 sec. while experiment showed the time to be 1·5 sec.

An experiment on a mild steel test piece 10 in. long between gauge points showed an amplitude of vibration of 0·0006 in., corresponding to the full swing of the beam. On the whole length of specimen the amplitude was about 0·0007 in. The big difference between the observed and calculated results is

due to lost motion in the parts and will be smaller the greater the leverage of the machine.

The manufacture of the Buckton machines is now in the hands of Messrs. W. & T. Avery Ltd., Birmingham.

Lever System of Multiple-lever Machine. The lever system of a multiple-lever machine is shown diagrammatically in Fig. 30. The straining screws pass through bearings in the base F and

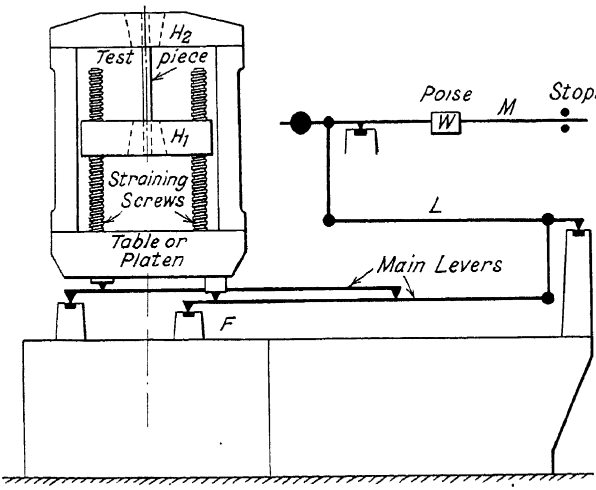


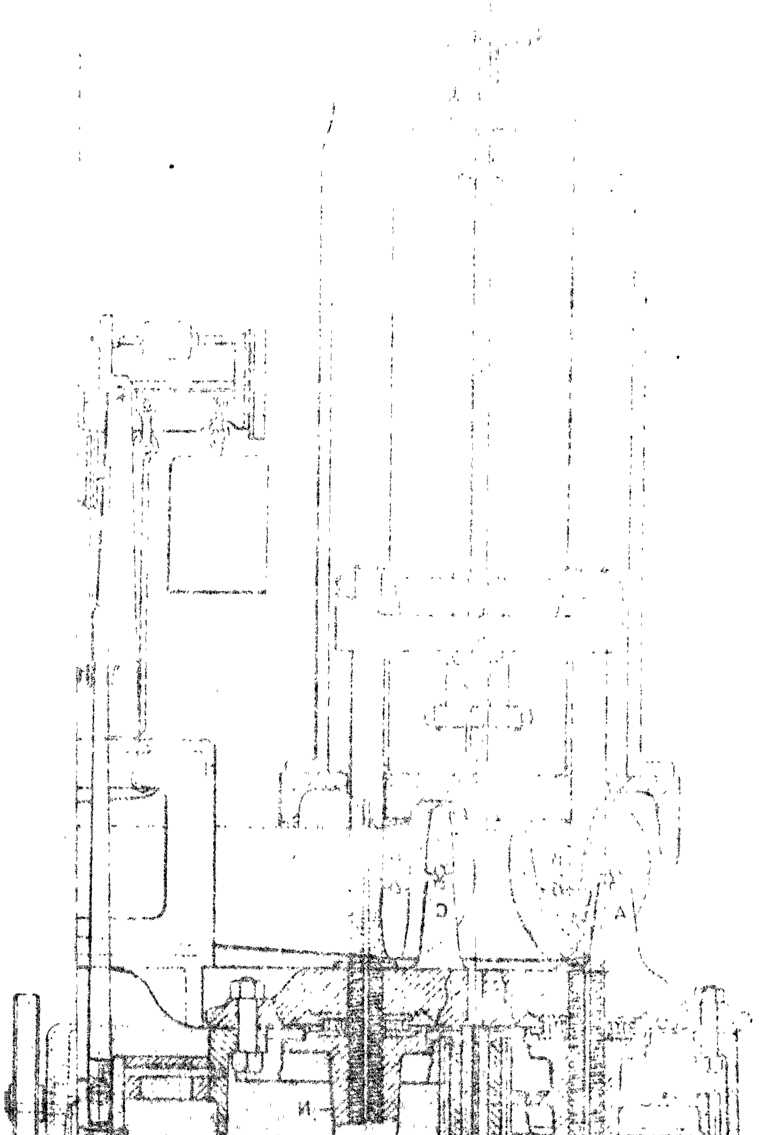
FIG. 30. LEVER SYSTEM OF MULTIPLE-LEVER MACHINE

rotate in bronze nuts in the pulling head H_1. The downward pull on the specimen is transmitted through the head H_2 to the platen resting on the main levers, and is then transmitted through the intermediate lever L to the steel yard M. Owing to the great leverage ratio, about 3 000 to 1 at the end of the steel-yard, the poise need weigh only a few pounds.

The testing speeds available vary with the size of the machine. In the case of a machine giving six pulling speeds they are approximately—

8·0 in. per min. for setting the head.
2·0 in. per min. for quick testing.
1·0 in. per min. for medium testing.
0·4 in. per min. for slow testing.
0·2 in. per min. for crushing tests.
0·1 in. per min. for slowest speed.

The Olsen Multiple Lever Machine is shown in Figs. 31 and 32. In this machine the straining screws work in rotating nuts


A
И

N and do not project above the pulling head. The space around the tension specimen is thus less restricted than would otherwise be the case, which is sometimes an advantage when an extensometer is used. A pit underneath the machine is needed to accommodate the straining screws. Two, three, or

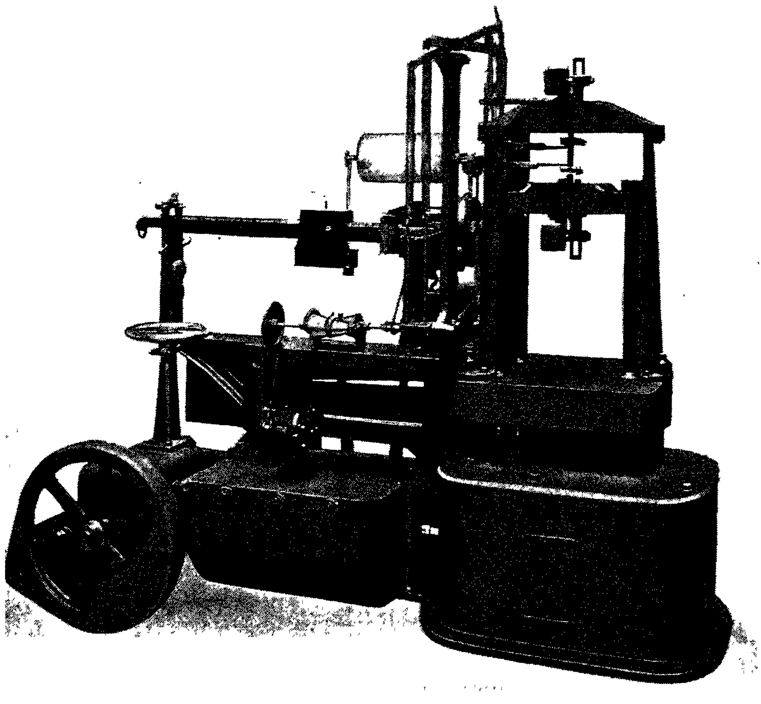


Fig. 32. Rear View of Olsen Multiple-lever Testing Machine
(Tinius Olsen Testing Machine Co.)

four screws may be employed and the nuts are of exceptional length in order to minimize wear.

The main levers rest on fulcra as indicated at *A* and *C*, Fig. 31, and support the platen on knife-edges as at *B* and *D*. The recoil of the system when the specimen fractures is taken by rubber buffers on the top corners of the platen. Three poise weights are provided, one for registering full load, one for half load, and one giving $\frac{1}{10}$ full load. An adjustable counterweight

enables the weight of the specimen and grips to be balanced before commencing a test.

An interesting feature is the autographic gear for recording the test and its automatic control. The autographic gear is shown in Fig. 33 and is entirely independent of the weighing system.

The clamps *B* are attached in position on the specimen by means of a jig or setting device. The calliper fingers *A* rest in contact with the clamps on the specimen. The elongation of the specimen is transmitted through the calliper fingers to the tubes *C* and *D* respectively. Relative movement between the tubes is then converted into rotation of the drum *R* by means of the thin metal band *L* fastened at *K*. This rotation provides the strain ordinate.

The screw on the steelyard controls the motion of the poise *V* through variable speed cones *Z* (Fig. 31) which are driven by an independent electric motor. The poise *V* moves the pen carriage *T* on its guide *Q* a corresponding amount by means of the cord *W* passing under pulley *VA* and over pulley *VB*. The combined motion of the pencil and drum develops the load-extension diagram.

In one type of recording gear the pencil is kept in a state of vibration by an electromagnet and traces out the record as a succession of dots the object being to avoid pencil friction. The speed cones permit the rate of travel of the weighing poise to be easily and quickly adjusted in order to produce a characteristic diagram under all conditions.

The method of using the autographic attachment is as follows.

The beam is balanced with the poise at zero and the clamps attached to the specimen as in Fig. 33. The hand lever *O* is latched down at *I*, which is the locking position, when the calliper fingers may be adjusted on their respective tubes to the required positions. The calliper fingers should be so placed as to be from half an inch clear to lightly touching both sides of each clamp *B*. In the tension test it is necessary that the upper calliper fingers be placed on the vertical tube *C* in front of the machine, while the lower calliper fingers are attached to the tube *D* at the rear.

The two tension springs *G* and *H* at the rear of the machine should be so placed that the heavier of the two springs will be at the position shown at *G*. The hand lever *O* is then unlatched from position *I* and raised to the extreme position where it

is latched at J, and at the same time the calliper fingers are guided to their respective clamps to ensure proper seating.

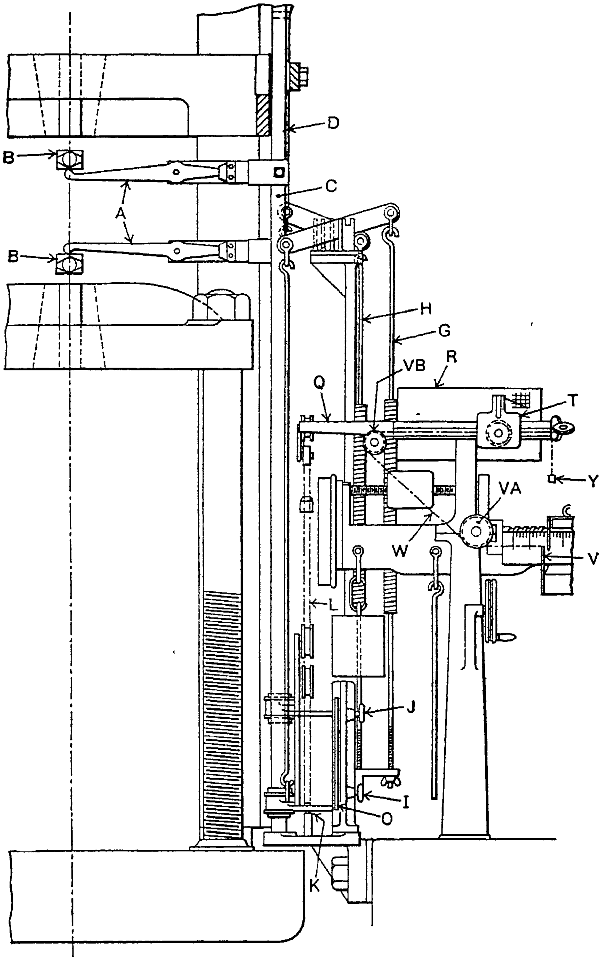


FIG. 33. AUTOGRAPHIC ATTACHMENT FOR OLSEN TESTING MACHINE
(*Machinery*)

The upper calliper fingers drop on to the lower clamp. If this action is not secured the spring tension on G and H should be adjusted.

leads to a small cylinder containing a piston *D*. This communication is always open and cannot be cut off by the valve *B*, so that the dynamometer is always loaded to the same pressure as exists in the straining cylinder. The outward movement of the piston causes the pendulum *P* to deflect and thus to

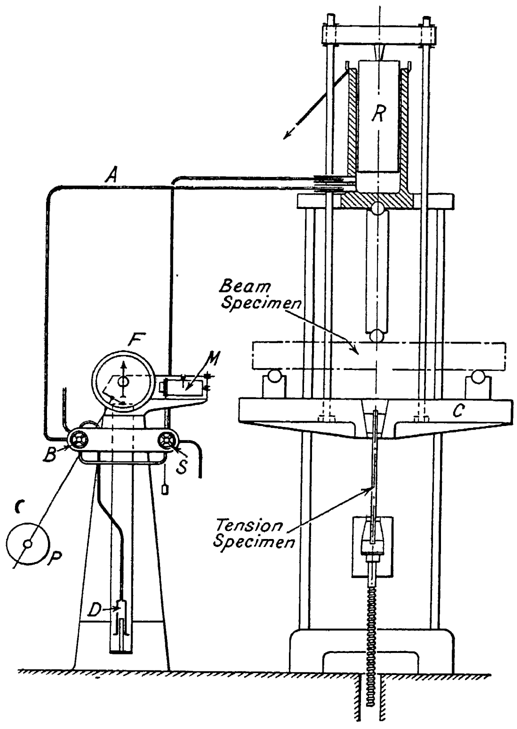


FIG. 34. DIAGRAMMATIC VIEW OF AMSLER TESTING MACHINE

indicate on the dial *F* the load exerted on the test piece. The drum *M* enables autographic records to be obtained.

The makers guarantee these machines to 1 per cent absolute accuracy for all loads above half the indicated maximum. All machines comply with the regulations laid down by the International Association for Testing Materials.

Calibration of Testing Machines. To calibrate a testing machine accurately is a matter of some difficulty. The really satisfactory way is to load the machine with dead weights

throughout its entire range. This is seldom possible after a machine has been installed and indirect methods have to be employed.

With single-lever machines the weight of the poise and the length of the short arm of the lever can be verified as follows—

The weight of the poise can be ascertained (*a*) by weighing either with a weighing machine suspended from a crane or by

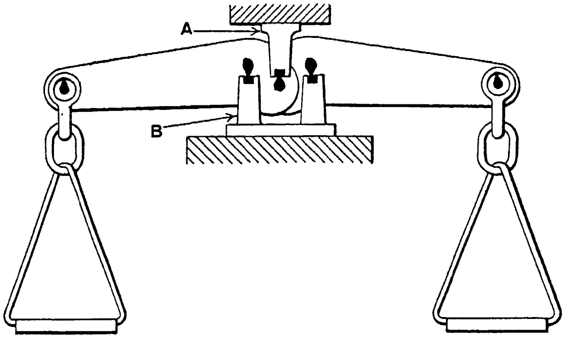


FIG. 35. ARRANGEMENT FOR CALIBRATING MULTIPLE-LEVER TESTING MACHINE
(*Machinery*)

removing the poise and weighing it on a platform machine; (*b*) by the method now to be described.

Balance the beam and adjust the vernier to zero. Hang a known weight w from the beam at a known distance l from the fulcrum. Restore balance by running the poise along the beam. If W is the weight of the poise and L the distance it is moved in order to restore balance, we have by moments

$$\mathrm{W} \times \mathrm{L} = w \times l$$

hence
$$\mathrm{W} = wl/\mathrm{L}.$$

To check the distance between the knife-edges, balance the beam and adjust the vernier to zero. Hang a heavy weight w in the shackle of the machine and move the poise forwards until equilibrium is restored.

If l_s is the length of the short arm of the lever, by moments

$$l_s = \mathrm{LW}/w$$

With multiple lever machines a method frequently adopted is to use proving levers. One form, for use in compression, is shown in Fig. 35. Two levers are arranged with their fulcra

on the part A which bears on the underside of the pulling head. The levers rest on the knife-edge seats in the casting B supported on the platen of the machine. Weights are placed in the scale pans at the ends of the levers and comparison made with the corresponding positions of the poise on the steelyard when the beam is in balance at the respective loads.

The machine is first balanced for the deadweight of the loading device before applying the calibrating weights. The load on the machine is calculated from the leverage ratio, which is usually 10 to 1 or 20 to 1.

Another method of calibration is to employ a standard steel bar and an accurate extensometer. The bar is first tested in a machine of known accuracy and its modulus of elasticity determined. A test is then made on the machine under consideration and the scale readings compared with those calculated by using the modulus of elasticity previously determined.

The American Society for Testing Materials now stipulates that machines up to 50 tons capacity shall be tested at the makers' works by applying dead loads throughout the entire range and the same stipulation applies to calibrating bars. As an alternative to a bar a steel ring may be used. Rings may be obtained fitted with a dial gauge when they are known as *Morehouse rings*. They are stated to be accurate to two-tenths of one per cent between one-fifth and full load capacity. Rings up to 300 tons capacity are obtainable. It is necessary to employ separate rings when calibrating a machine on both rising and falling loads on account of elastic hysteresis in the steel.

A modification of the foregoing method is found in the *Standardizing boxes* made by Messrs. Amsler, Fig. 36. Boxes are made either for tension or compression loading and also in a combined form which permits the box to be used in tension or compression as desired.

The box is a hollow steel cylinder A and is filled with mercury. At one end and on one side of the box the hollow space communicates with a capillary tube C, which terminates in a small bulb. On the other side is a stem provided with micrometer screw plunger E. By displacing with the plunger some of the mercury in the box the end of the mercury column in the capillary tube may be brought to the edge of a datum mark D. When the box is compressed it shortens and its internal capacity

diminishes. This shortening ejects into the capillary tube a volume of mercury equal to the diminution in volume of the interior of the cylinder. The amount of mercury expelled is a measure of the shortening of the cylinder and consequently of the load exerted upon it. The amount so expelled is measured by the micrometer screw.

On commencing the test the micrometer handle is turned until the end of the mercury column in the tube is at the edge

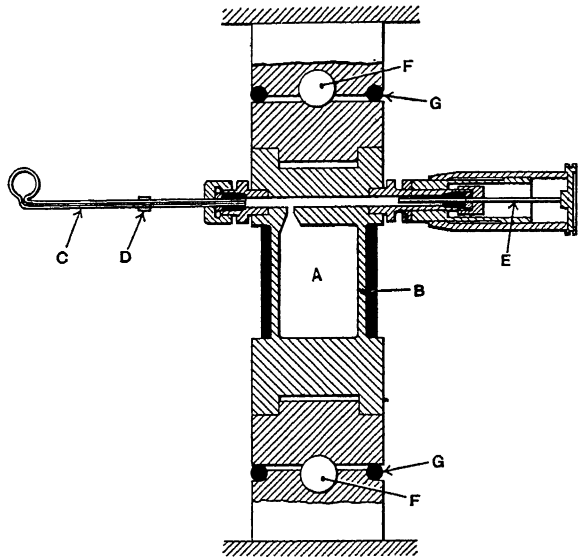


FIG. 36. AMSLER STANDARDIZING BOX
(*Machinery*)

of the datum mark and the scale reading noted. When a load is applied to the box, mercury is forced into the capillary tube. The micrometer is again turned to bring the mercury to the zero mark and the new scale reading observed. The difference between the two readings corresponds to the compressive load on the box.

The mercury meniscus is always restored to the same point before each reading of the micrometer. The results are thus independent of variations in the diameter of the glass tube.

The box is insulated at B to prevent rapid changes of temperature from causing an alteration of the mercury column. To prevent the standardizing box from being stressed eccentrically

it is necessary to support the box so that it can adjust itself freely to the axis of the machine. It is also necessary to take precautions to ensure that the box does not upset in case one of the compression plates is not guided but can yield laterally. If neither compression plate can yield laterally it is advisable to apply the load through balls *F* placed between the seatings. The rubber ring *G* prevents the box from falling over after it is completely unloaded.

If one of the compression plates can yield laterally, as in the Amsler machine, it is necessary to dispense with one of the steel balls and the accompanying seating and to place the standardizing box on the fixed compression plate of the machine while retaining the other ball and its seating.

The loads recorded by the weighing apparatus of the machine being calibrated are compared with a scale of loads engraved on the box.

Compression boxes are made for machines up to 500 tons and tensile boxes for machines up to 300 tons capacity.

The boxes can be used equally well with vertical or horizontal machines and each box is guaranteed to an accuracy of within $\frac{1}{2}$ per cent for all loads above one-tenth the capacity of the box.

Calibration of a 600 000 lb. Riehle Machine. Another method of calibrating a machine throughout its entire range consists in the use of dead weights together with a calibrating bar. A 600 000 lb. Riehle machine was calibrated by this method at the University of Illinois by using two 20 000 lb. weights and a steel bar, 19 ft. long and 4 in. diameter, carrying a strainmeter of 100 in. gauge length.

One complete division of the micrometer dial of the strainmeter represented 0·00005 in. stretch.

The procedure was as follows. After balancing the machine at zero load the weights were lowered on to the table by means of jacks. The machine was again balanced and the error at 20 000 lb. noted.

With the poise weight at the position found, the weights were lifted off the table until the beam was once more in balance. The reading of the strain gauge was noted and the weights again lowered on the table. This dead load relieved the pull in the bar to a small extent and caused the strainmeter reading to fall slightly. Further load was applied through the straining gear to bring the meter back to its former reading. The load on the

machine now consisted of 20 000 lb. dead load and 20 000 lb. pull in the bar, making a total of 40 000 lb.

After balancing the beam and determining the error at this load the procedure was continued in increments of 20 000 lb. until full load was reached.

With this method it is not necessary to evaluate the modulus of elasticity of the bar. All that is required is that a loading should be reproduced accurately. Any error in a reading is carried forward throughout the series, but some counter-balancing of such errors should occur as the calibration proceeds.

The maximum error in the machine tested, 0·15 per cent, occurred at 50 000 lb. load. At 300 000 lb. the error was negligible and at full load was less than 0·1 per cent.

Jakeman's Method of Calibration. The method of calibrating a single-lever testing machine described on page 56 has been elaborated by Jakeman. Two pieces of angle iron each with a V-groove milled along its whole length and having a hole drilled at its centre O, Fig. 37 (*a*) and (*b*), are bolted to the ends of the steelyard. The angle irons are of such a length that the lines *AA* and *BB* are clear of projections on the machine. The V grooves provide seatings for supporting the scale pans P_1 and P_2, Fig. 38. The stem of each scale pan is attached to a short length of steel rod which rests in the groove in the angle iron, Fig. 37 (*b*).

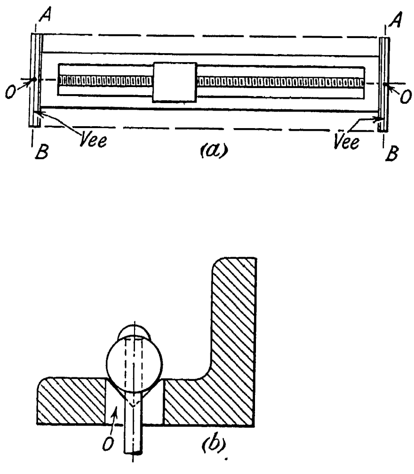


FIG. 37. JAKEMAN'S METHOD OF CALIBRATION

(*a*) Attachment of angle irons to weigh-beam.
(*b*) Method of suspending scale pans.

The lengths *AA* and *BB* are measured by means of an accurate steel tape, the mean of the two measurements giving the distance L between the suspensions P_1 and P_3.

For the calibration two or three 100 lb. weights, a number of 2 lb. weights, and a set of weights from 1 lb. to 0·001 lb. are needed.

An additional scale pan P_3 is suspended from the loading shackle of the machine.

To determine the weight of the poise (w).

As many of the 100 lb. weights as convenient are placed on the pan P_1 and the lever balanced by moving the poise. The 100 lb. weights are now moved from P_1 to P_2 and the distance S

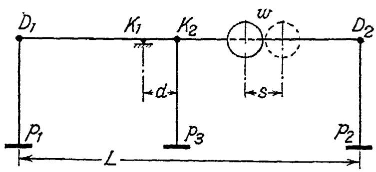


FIG. 38. JAKEMAN'S METHOD OF CALIBRATION

through which the poise has to be moved to secure balance is measured.

If W be the total weight moved from P_1 to P_2 then

$$w = (\mathrm{W} \times \mathrm{L})/S$$

To determine the knife-edge distance (d).

The small weights are all placed on P_2 and the beam balanced. The poise is then wedged in position so that it cannot move. The large weights are placed on P_3 and the beam balanced by moving weights from P_2 to P_1. If m is the sum of the weights removed from P_2 to P_1 and W′ the total weight placed on P_3,

$$d = (\mathrm{L} \times m)/\mathrm{W}'$$

The length l on the scale representing 1 ton can be checked without measuring the distance L, for

$$l = \frac{d \times 2\,240}{w} = \frac{\mathrm{L}m}{\mathrm{W}'} \times \frac{2\,240S}{\mathrm{WL}}$$

The method may be used to check the knife-edge distance under load using a large weight on P_3. In order to obtain sufficient sensitivity, the motion of the lever must be appreciable when moving the small weights from P_2 to P_1. Jakeman suggests that a load of the order of 50 tons should be applied through a railway coupling. The ram must be blocked in position if the machine is hydraulically loaded.

As the sensitivity of the machine is less under these conditions several observations should be taken.

CHAPTER IV

TENSION AND BENDING TESTS

The Load-extension Diagram for Mild Steel. If a bar of wrought iron or mild steel be tested in tension a graph showing the relation between the load and extension, or between stress

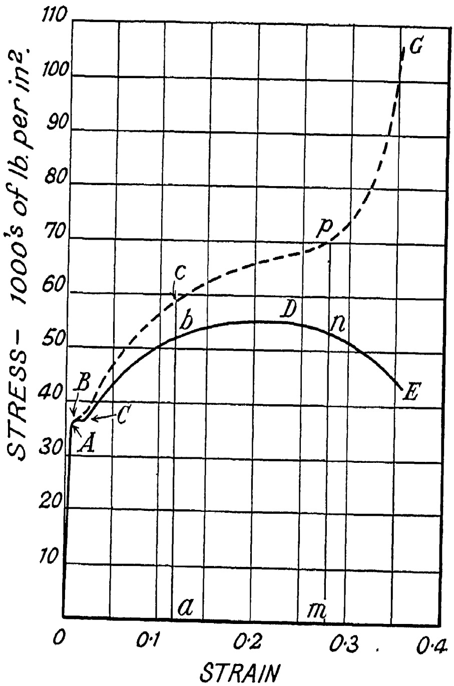


Fig. 39. Stress-strain Curve for Mild Steel in Tension

and strain, will be, generally, of the form shown by the full line curve in Fig. 39.

The extension will be found to be proportional to the load over a considerable range OA, after which, as the load is gradually increased, the graph will deviate from a straight line and for very little greater load the extension will proceed rapidly without any further rise of load occurring. This is indicated by the portion BC of the curve. Actually, a sudden

drop of the load occurs at this stage owing to the rapidity with which the specimen extends. The reduction of load is made visible by a drop of the weighbeam of the testing machine, or, if an autographic record be taken, the graph will usually show a drop at this point to an extent depending on the sensitiveness of the recording gear. A record taken on a commercial machine may show a reduction of 5 per cent of the load, but with special apparatus under laboratory conditions a drop of 27 per cent has been recorded.

The point B at which this sudden yield occurs is termed the *yield point* of the material.

In wrought iron and mild steel the limit of proportionality and the elastic limit practically coincide, but this is not generally so with other materials or with a material that has been overstrained.

With further application of the straining action the test piece will withstand still higher loads, but stress and strain are no longer found to be proportional. The graph follows some such line as CD until the maximum load is reached. During this plastic stage the cross-section of the material diminishes in about the same proportion as the length increases, and on passing the maximum load a sudden local stretching takes place over a short length of the test piece and a waist or neck is formed.

This greatly reduced area is insufficient to sustain the load and it will be found that, to preserve balance of the steel yard, the poise must be run back towards zero.

From D the curve falls until fracture of the test piece occurs, indicated by the point E.

The actual stress at fracture is, however, much higher than that corresponding to the maximum load since it is given by

$$\frac{\text{Load}}{\text{Actual area of cross-section}}$$

and the proportionate reduction in section is considerably greater than the reduction in load. The graph, therefore, exhibits only a nominal indication of the stress at any point during the plastic stage.

It is difficult to obtain the complete load-extension curve by direct observation but, supposing the curve obtained and plotted as a stress-strain diagram, the corrected curve, shown dotted in the figure, may be plotted as follows.

at the centres of the respective inch divisions, a curve drawn through the plotted points will be fairly symmetrical about the position of fracture (Fig. 41). It will be noted that the extension is nearly constant except in the immediate vicinity of the fracture.

The percentage extension, a criterion of ductility, depends upon the gauge length adopted. Fig. 42 is obtained by plotting

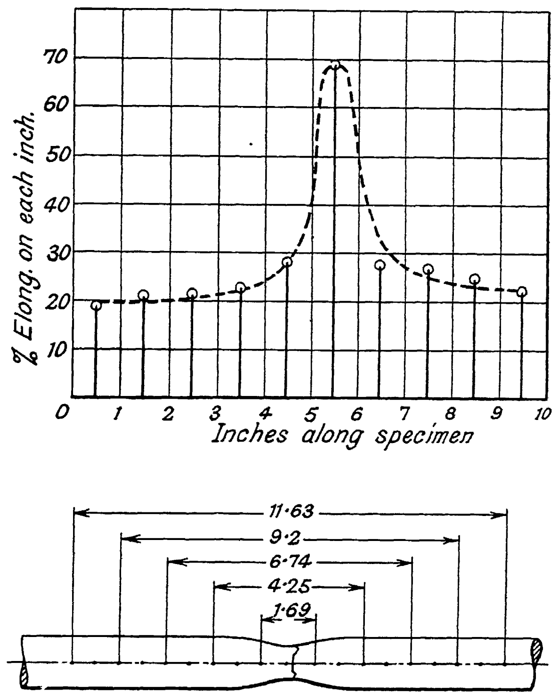


FIG. 41. VARIATION OF UNIT-EXTENSION ALONG GAUGE LENGTH

the percentage extensions observed in a test on a mild steel bar on gauge lengths of 1, 3, 5, 7, and 9 in., all of which include the fractured section. The extension on a gauge length of 2 in. is 51 per cent, while on a gauge length of 8 in. it is only 26 per cent. This shows clearly the necessity for stating the gauge length when specifying the percentage elongation. Both the percentage elongation and the percentage reduction in area of cross-section are regarded as criteria of ductility.

The total elongation of the fractured test piece is made up

of the local extension in the region of fracture together with the more uniform extension over the remainder of the gauge length.

The local extension occurs mainly after the maximum load is reached and at the point where the neck or waist forms. The length over which the local extension exists depends to some extent upon the area of the cross-section and is approximately proportional to the square root of that area. Unwin has shown that the relation between the percentage elongation e and the dimensions of the test piece may be represented with fair accuracy by an equation of the form

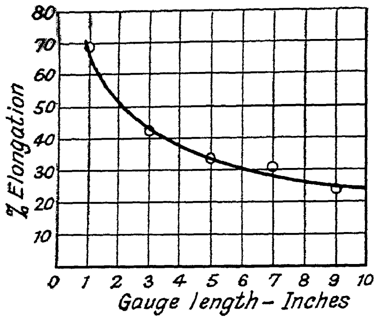


FIG. 42 PERCENTAGE ELONGATION VARIES WITH THE GAUGE LENGTH

$$e = (c\sqrt{a})/l + b$$

where $(c\sqrt{a})/l$ represents the local extension and b the general extension; a being the area of cross-section and l the gauge length.

Average values for the constants c and b are—

Material	Values of the Constants	
	c	b
M.S. plates, not very thick	70	18
Gun metal (cast)	8·3	10·6
Rolled brass	101·6	9·7
Rolled copper	84	0·8
Annealed copper	125	35

Proportions of Test Pieces. The percentage elongation depends also on the form of the test piece and it was suggested by Barba in 1880 that for strictly comparable results, test pieces should be similar.

In the case of cylindrical test pieces this leads to the relation l/d = a constant. In Germany this ratio is usually taken as 10, which makes $l = 11{\cdot}3\sqrt{a}$, where a is the cross-sectional area in cm.^{2} and l is the length in cm.

The values adopted by the British Standards Institution are—

d (in.)	a (in.^{2})	l (in.)
0·564	0·25	2
0·798	0·5	3
0·977	0·75	3·5

The above values correspond approximately to $l = 4\sqrt{a}$.

In America the relation $l = 4{\cdot}5\sqrt{a}$ is adopted.

In the testing of plates, where test pieces are rectangular in section, adherence to the similarity law is attended with difficulties and hence considerable departure from the theoretic law is made.

For instance, the B.S.I. test pieces for plates are of the form shown in Fig. 43, and to lessen the cost of production three

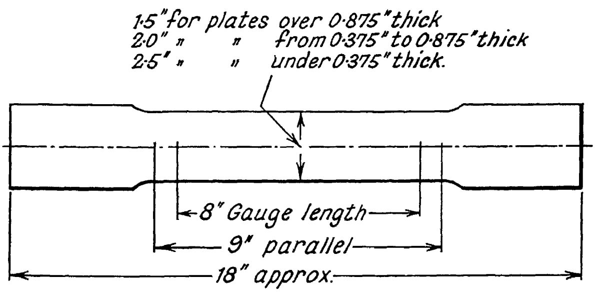


FIG. 43* FORM OF BRITISH STANDARD TEST PIECES FOR PLATES

widths only are specified. The width of 2 in. was selected for the sake of uniformity and as the percentage elongation was found to be less for thick than for thin plates the other widths were chosen in order to slightly favour the thicker plate.

To secure the same percentage elongation with the same material, for round specimens the gauge length is made eight times the diameter and the parallel portion nine times the diameter.

The specification for cylindrical test pieces for forgings,

* Abstracted by permission from British Standard Specification 18—*Forms of Tensile Test Pieces*, copies of which can be obtained gratis from the British Standards Institution, 28 Victoria Street, London, S.W.1.

axles, etc., permits various lengths and diameters to be employed. Particulars of these are given in Table III, which has reference to Fig. 44.

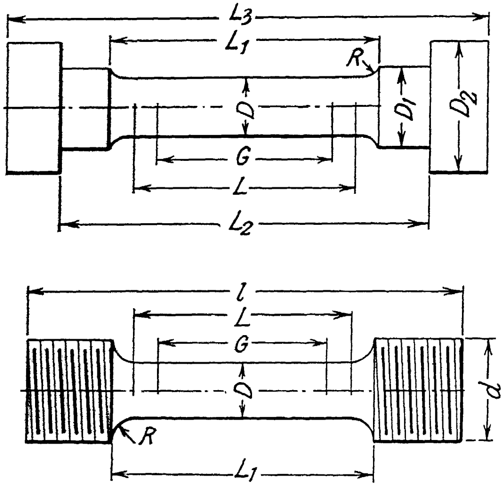


FIG. 44.* FORMS OF CYLINDRICAL TEST PIECES

The form of the ends of the test piece is a matter of some choice. If an extensometer is to be used on the specimen the length between the grips may need to be increased to accom-

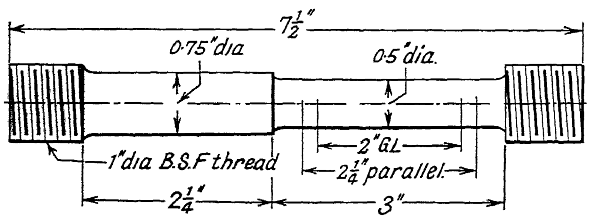


FIG. 45. ALTERNATIVE FORM OF TEST PIECE

modate the instrument. To receive the Ewing extensometer, for instance, the length between the screwed portions of the test piece must not be less than 4½ in. for a gauge length of 2 in.

Another form of test piece is shown in Fig. 45. The extra

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TABLE III

Dimensions of Cylindrical Test Pieces

Diameter D	Area of Cross-section	Gauge Length G	Length of Parallel L	Length between Shoulders		Overall Length		D_1	D_2	d	Radius R
				L_1	L_2	L_3	l				
In. 0·564	In.^{2} 0·25	In. 2·0	In. 2·25	In. 3·0	In. 4·25	In. 5·25	In. 4·5	In. 0·75	In. 1·25	$\frac{3}{4}$ B.S.F.	As large as con-venient
0·798	0·50	3·0	3·375	4·0	5·25	6·5	6·5	1·0	1·5	$1\frac{1}{4}$ B.S.W.	
0·977	0·75	3·5	4·0	5·0	6·25	8·0	8·0	1·25	1·75	$1\frac{1}{2}$ B.S.W.	

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length of plain portion permits a Brinell test to be made on specimen.

The position of the fracture on the test bar will influence to some extent the percentage elongation. If fracture occurs within the middle third of the gauge length the resulting value of the percentage elongation may be accepted, but if outside this limit a correction should be made if it is desired to obtain strict comparisons.

Timoshenko gives the following method of making the correction.

If the gauge length has been divided into n parts of equal length before testing and if $l_t =$ the total extension required and $x =$ the extension on the division containing the fracture; then, if fracture occurs outside the middle third and the number of divisions n is even, the measurements being made on the longer part of the test piece, the extension

$l_t = x +$ extension on $(n/2 - 1)$ divisions $+$ extension on $n/2$ divisions.

If n is odd

$l_t = x +$ extension on $\frac{1}{2}(n - 1)$ divisions $+$ extension on $\frac{1}{2}(n - 1)$ divisions

where the measurement is again made on the longer part of the test piece.

According to a German rule a test piece of 200 mm. gauge length is divided into 20 equal parts. If fracture occurs outside the middle third the division marks are numbered on either side of and away from the fracture, 0 to n on the shorter portion, and 1 to 10 on the longer portion. Then the required extension

$l_t =$ length of fractured division $+$ length between 0 and $n +$ length between 1 and 10 $+$ length between n and 10 on the long portion.

For a gauge length of 100 mm. the gauge length is divided into ten equal parts; five being then substituted for ten in the foregoing rule.

Certain types of extensometer are pinched on to the specimen by means of pointed set screws. The small centre pops formed in the test piece at the gauge points tend to cause fracture to occur there rather than in the middle of the gauge length. Should fracture occur at a gauge point the test should be

disregarded. A test is generally considered to be satisfactory if fracture occurs within the gauge length and not nearer to a gauge point than a distance equal to $\sqrt{}$(area of cross-section).

To prevent fracture at a gauge point, brittle materials are tapered two- or three-thousandths of an inch from the gauge point towards the centre, the middle portion of the gauge length being left parallel.

Forms of test pieces for brittle materials are shown in Fig. 46.

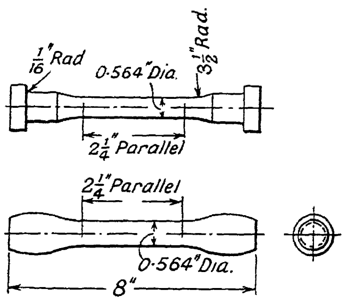

FIG. 46.* FORMS OF TEST PIECES FOR BRITTLE MATERIALS

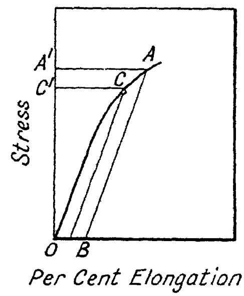

FIG. 47. METHOD OF TESTING BASED ON PROOF STRESS

Proof Stress. Determination of the Modulus of Elasticity. The terms limit of proportionality, elastic limit and yield point are often used indiscriminately. To avoid confusion, brought about by careless expression, the term *proof stress* is frequently employed in specifications, especially when the material in question shows no well-defined yield point.

A given intensity of stress is specified to be applied to the test piece for a definite time, and on removal of the stress, the permanent set of the material must not exceed a predetermined amount: 1 per cent, or in some cases 0·5 per cent, of the elongation under the given load.

For example, if OB, Fig. 47, represents the predetermined elongation and BA is drawn parallel to the elastic part of the stress-strain curve to intersect the graph in the point A corresponding to the given stress OA', then, if the proof stress OC'

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gives a point C below A on the curve, the permanent set will fall within the required limit.

The value of the modulus of elasticity is sometimes obtained in a similar way by applying a load which will just produce a permanent set, and noting the extension produced. The load is then nearly all removed and the extensometer reading again observed. The value of the applied stress divided by the difference between the extensometer readings, expressed as a strain, is taken as the modulus of elasticity. The merit of the test lies in the rapidity with which it can be carried out.

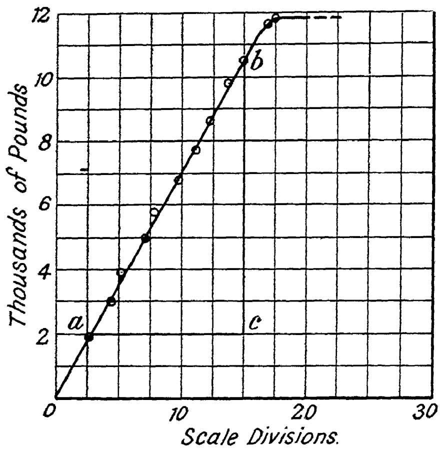


FIG. 48. USE OF GRAPH IN DETERMINING YOUNG'S MODULUS

The modulus of elasticity, however, is preferably obtained from a graph plotted from the recorded loads and extensions as in Fig. 48, which is from a test on a specimen 0·56 in. diameter and 2 in. between gauge points from a steel casting. The limit of proportionality, 10 300 lb., corresponds to a stress of 19·56 tons per in.^{2}

$$\text{The modulus of elasticity E} = \frac{\text{load} \times \text{length}}{\text{area} \times \text{extension}}.$$

The value of one scale division on the extensometer used was 1/5 000 in. From the graph, taking two points a and b on the straight portion, we find that the extension ac scales 12·1 instrument divisions and the load cb scales 8 500 lb.

The extension is

$$12{\cdot}1 \times \frac{1}{5\ 000} = 0{\cdot}00242 \text{ in.}$$

Hence

$$E = \frac{8\ 500 \times 2}{(\pi/4) \times (0{\cdot}56)^2 \times 0{\cdot}00242}$$

$$= 28\ 500\ 000 \text{ lb. per in.}^2$$

In this example the values of the load and extension might have been read off from one point on the graph as this passes through the origin. Such is not always the case, as corresponding to zero reading of the instrument a slight load on the specimen may have to be imposed in order to ensure that the grips are biting if specimens are being tested in wedge grips. As a fair curve drawn the plotted points may not, in general, pass through the origin, the slope of the graph should always be determined by taking two selected points on the line.

Form of Report for Tensile Tests. A form of report for tensile tests is given below.

REPORT OF TENSILE TEST FOR . ...

Date of Test ....

Particulars of Specimen....................*Flat Machined Steel*..... ....

No. of Specimen ....

Mark of Owner ....

Original dimensions—		
Length between gauge points		8 in.
Breadth		1·546 in.
Thickness		0·238 in.
Diameter		—
Original area of cross-section	$1{\cdot}546 \times 0{\cdot}238 =$	0·368 in.²
Limit of proportionality—		
Total load		— tons
Stress		— tons per in.²
Yield—		
Total load		9·3 tons
Stress		25·27 tons per in.²
Maximum load		14·46 tons
Ultimate stress		39·74 tons per in.²
Load at rupture—		
Total		— tons
Stress		— tons per in.²

TABLE IV

STANDARD TEST BARS FOR TRANSVERSE TESTS

Test Bar	Diameter (in.)	Overall Length (in.)	Main Cross-sectional Thickness of Casting Represented (in.)
S	0·875	15	Not exceeding ¾
M	1·2	21	Over ¾ and not exceeding 2
L	2·2	21	Over 2

Bars which exceed the standard diameters by more than 0·1 in. must be turned down to the standard dimensions. Certain minimum test requirements have to be met if bars vary

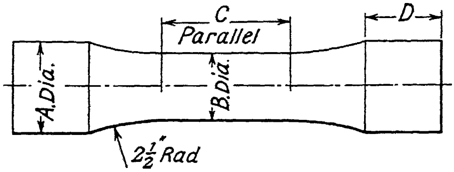


FIG. 49.* TENSILE TEST PIECE FOR CAST IRON

more than 0·1 in. from the standard dimensions. (See B.S.S. No. 321.) The form of test piece for tensile tests is shown in Fig. 49, to which Table V refers. The corresponding loadings are given in Table VI.

TABLE V

STANDARD TEST PIECES FOR TENSILE TESTS

Test Bar	Dimension (in.)				Main Cross-sectional Thickness of Casting Represented (in.)
	A	*B*	*C*	*D*	
S	0·875	0·564	2	1	Not exceeding ¾
M	1·2	0·798	2	1	Over ¾ and not exceeding 2
L	2·2	1·785	2	1	Over 2

Test bars are to be cast as parallel bars of the diameter given in *A* and machined to the dimensions *B* and *C*.

* Abstracted by permission from British Standard Specification 18—*Forms of Tensile Test Pieces*, copies of which can be obtained gratis from the British Standards Institution, 28 Victoria Street, London, S.W.1.

TABLE VI

VALUES OF THE MODULUS OF RUPTURE FOR BARS UNDER STANDARD CONDITIONS

Test Bar	Distance between Supports (in.)	Transverse						Tensile	
		Minimum Load in lb.		Minimum Deflection (in.)		Modulus of Rupture Tons per in.^{2}		Minimum Tons per in.^{2}	
		Grade		Grade		Grade		Grade	
		I	II	I	II	I	II	I	II
S	12	1 185	960	0·12	0·1	24·1	19·6	12	10
M	18	1 950	1 600	0·15	0·12	23·1	18·9	11	9
L	18	10 000	8 950	0·12	0·1	19·2	17·2	10	9

Modulus of Rupture. The transverse strength of cast iron is often gauged by the *modulus of rupture*. This quantity is the ratio of the maximum bending moment at failure to the modulus of the section (page 9). The modulus of rupture is thus the greatest stress that would have obtained had the material obeyed Hooke's law up to the point of fracture. If this condition existed the value of the maximum stress would be independent of the shape and size of the section. However, since the linear relationship between stress and strain ceases long before the point of fracture is reached, the modulus of rupture must be regarded "as a convenient way of expressing the results of transverse tests without giving full details of the bar dimensions."

If L is the distance between the supports in inches;
W the load applied at the centre of the span in tons;
Z the modulus of the section;
f the maximum stress induced, tons per in.^{2}

then
$$f = WL/4Z$$
and if W is the load when fracture occurs the corresponding value of f is the modulus of rupture.

$$\text{For a rectangular bar } f = \frac{\text{breadth} \times \text{depth}^2}{6}$$

$$\text{For a round bar } \quad f = 0{\cdot}0982\ (\text{diameter})^3$$

Values of the modulus of rupture for bars under standard conditions are given in Table VI. With the older standard it was usual to specify that on a 36 in. span the bar should sustain

a load of 30 cwt. at the centre with a deflection of not less than 0·625 in. The modulus of rupture is affected by the method and conditions of casting and by the shape of the section. Rough bars are stronger than planed bars. With bars of similar proportions the modulus is lower as the section is larger. A wide bar gives a higher and a deep bar a lower modulus of rupture.

When a test bar is not of the standard dimensions the load that would produce the same stress in a bar of the same material of standard dimensions is termed the *equivalent load*. The equivalent load is calculated by the usual theory.

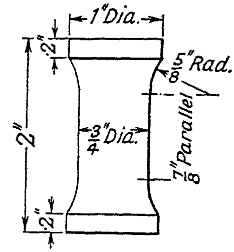


FIG. 50. COMPRESSION TEST PIECE FOR CAST IRON

For a bar of rectangular section, if W be the actual load on a bar of breadth b and depth d, the load W′ that would produce the same stress in a standard bar of breadth b' and depth d' is given by

$$W' = Wb'd'^2/bd^2$$

The load W^1 that would produce the same deflection as in the given beam is from the relation

$$\delta = Wl^3/48EI = Wl^3/4Ebd^3$$

obtained as $$W_1 = W(b'd'^3/bd^3).$$

But $$W_1 = W'\frac{bd^2}{b'd'^2}\cdot\frac{b'd'^3}{bd^3} = W'\frac{d'}{d}$$

and hence is not equal to the load required to produce the same stress.

The deflection produced by the "equivalent" load in the standard bar is

$$\delta' = \frac{W'bd^3}{b'd'^3W}\,\delta = \frac{b'd'^2}{bd^2}\cdot\frac{bd^3}{b'd'^3}\,\delta = \frac{d}{d'}\,\delta$$

This relation holds equally for bars of round section.

Crushing Tests. Crushing tests on cast iron are made on cylinders or prisms in which the ratio of the height to the least lateral dimension is from 1 to 3. The crushing strength varies from 30 to 50 tons per in.^{2} A suitable form of test piece is shown in Fig. 50.

Tests in direct shear show that the shearing strength of cast iron is somewhat greater than its tensile strength; generally from 10 to 20 per cent. When tested in torsion a round cast

iron bar fractures along a helix of 45° where the tensile stress is greatest.

Bend Tests. Bend tests on wrought iron and mild steel are carried out in order to determine the ductility. Bars are bent double or through a specified angle about a given radius. Reference should be made to various British Standard Specifications. The method of making the bend test is shown in Figs. 64 and 65, pages 85 and 86. The test as usually carried out is qualitative rather than quantitative but it has been suggested that its usefulness might be extended and that it might, to some extent, replace the tensile test.

CHAPTER V

TESTING MACHINE ACCESSORIES

Necessity for Loading Test Pieces Axially. Accurate results in tensile testing cannot be obtained unless test pieces are properly gripped. It is essential that the line of resultant pull should coincide with the axis of the specimen. The effect of eccentric loading can be gauged from a simple example. Suppose a pull of 10 tons to be applied to a test piece whose section is, say, 1 in. $\times$ $\frac{1}{2}$ in., and that the line of pull is displaced 0·01 in. from the axis towards the short side of the section. In addition to the direct pull there will be a bending moment of $10 \times 0{\cdot}01 = 0{\cdot}1$ ton-in. The stress due to bending is then

$$f = \frac{M}{Z} = \frac{0{\cdot}1 \times 6}{0{\cdot}5 \times 1^2} = 1{\cdot}2 \text{ tons per in.}^2$$

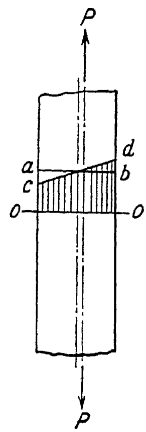


FIG. 51. STRESS VARIATION OVER SECTION OF TEST PIECE UNDER ECCENTRIC LOAD

The stress distribution across the section, instead of being represented by a line *ab*, Fig. 51, whose ordinate corresponds to 10 tons per in.^{2} will be represented by *cd*, the maximum stress being 11·2 tons per in.^{2} and the minimum stress 8·8 tons per in.^{2} As the load is increased the material commences to yield first on the side where the stress is greater, and finally tears from edge to edge. With ductile materials the elastic limit and yield point, calculated as total load divided by area of cross-section, are both lowered if the loading is eccentric, but the ultimate strength appears to be little affected. The ultimate strength of brittle materials is, however, greatly affected by eccentric loading.

Grips and Shackles. The simplest method of gripping flat test pieces is by means of a pin passed through a hole drilled in each end of the test piece. The pins are supported in shackles in the testing machine.

A common method is to use wedge grips. The grips are simply wedges with serrated faces and are held in a conical hole in the crosshead of the machine. Wedges for flat specimens sometimes have rounded faces as in the Riehle Patent Grip. Forms

of wedges for rounds, flats, and squares are shown in Fig. 52 (*a*), (*b*), (*c*). The wedges should sit fair in the holes in the cross-heads and should not project too far. If the wedges pull far

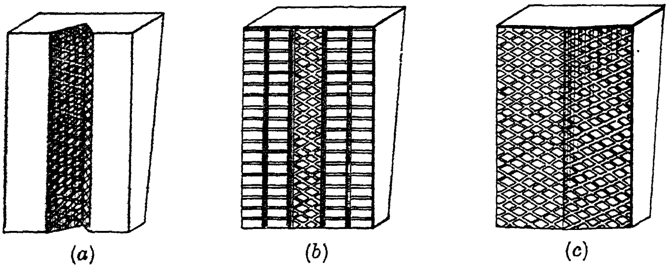


FIG. 52. WEDGE GRIPS FOR FLAT, ROUND, OR SQUARE SPECIMENS
(*Tinius Olsen Testing Machine Co.*)

below the cross-head they are liable to break and the cross-head holes tend to enlarge.

The test piece should be gripped along the whole length of the wedge. Figs. 53 (*a*) and (*b*), show how the wedges should *not* be arranged. The correct method of gripping is shown in Fig. 54. In Fig. 55 is shown a form of ball and socket liner which aids in securing alignment of the test piece.

Round specimens are often provided with a head or with screwed ends. Several methods of holding the specimens are

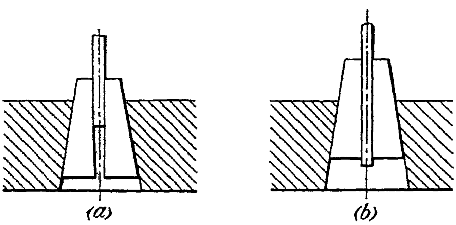


FIG. 53. INCORRECT METHODS OF GRIPPING TEST PIECES

in use which aim at securing axial loading. Fig. 56 shows three forms made by Messrs. Olsen. In (*a*) the spherical seat is in the wedge piece. Tool-steel bushes are provided to receive the specimen and either headed or screwed specimens may be accommodated. In (*c*) the holders are provided with a spherical adjustment and are for use with a 0·505 in. diameter headed specimen of aluminium or brass in machines of 10 000 to 20 000 lb. capacity.

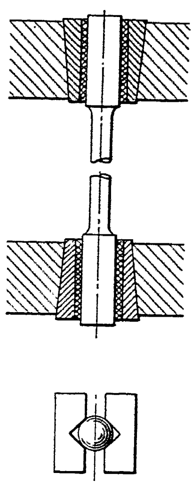


FIG. 54. CORRECT METHOD OF GRIPPING TEST PIECES

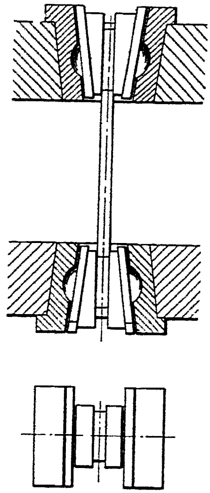
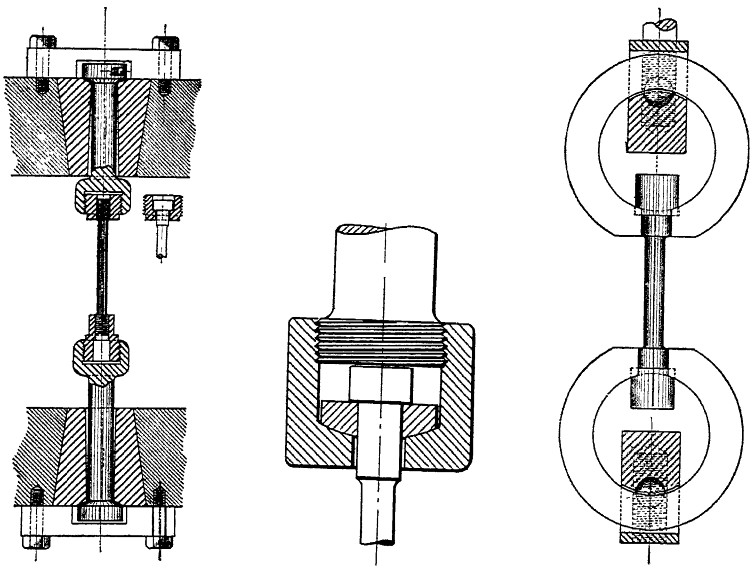


FIG. 55. BALL AND SOCKET WEDGE GRIPS



(a) (b) (c)

FIG. 56. HOLDERS FOR TENSION SPECIMENS WITH SCREWED OR SHOULDERED ENDS

(*Tinius Olsen Testing Machine Co.*)

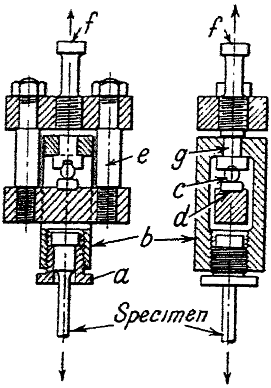


FIG. 57. ROBERTSON'S AXIAL LOADING SHACKLE

A special form of shackle has been devised by Robertson to secure true axial loading, Fig. 57. The specimen is held by screwed split collars *a*, in a socket *b*. The pull is taken on the ball *c* resting on the hard steel seat *c*. The frame *e* is provided with an extension piece *f*, for connection to the grips of the testing machine. The hole for the ball socket *g* and the threaded hole for the split collar should be machined at the same setting in order to secure alignment.

Fig. 58 shows a grip for specimens of thin sheet metal. Another form is shown in Fig. 59. The shackle consists of two steel plates supported on a pin in the head *b* which is screwed into the holder in the pulling head of the machine.

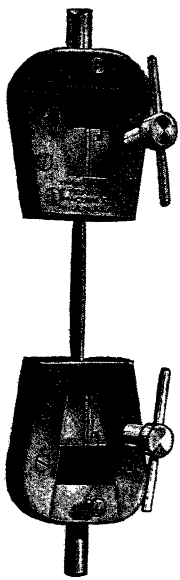


FIG. 58. WEDGE GRIPS FOR SHEET METAL TEST PIECES
(*Tinius Olsen Testing Machine Co.*)

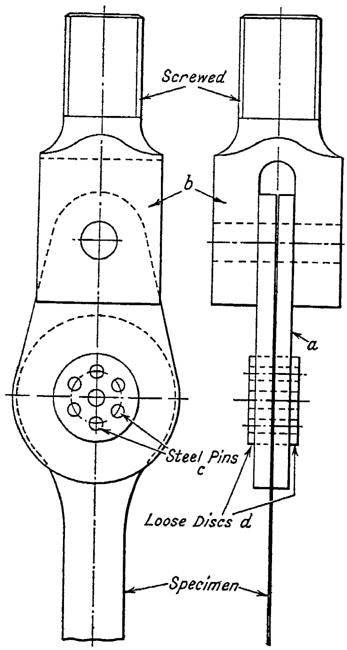


FIG. 59. SHACKLES FOR SHEET METAL TEST PIECES

The plates are drilled to receive two loose discs *d*. The disc and the sheet metal specimen are drilled to receive steel pins *c* which transmit the pull.

Testing of Ropes and Chains. Ropes are somewhat troublesome to test owing to possible damage to the rope at the grip. One method consists in casting a conical head at each end of the test piece. The ends of the rope are unwound for several inches depending on the size of the piece, being first bound up to prevent further separation of the wires along the rope. The wires are then splayed out,

FIG. 60. METHOD OF CAPPING WIRE ROPES

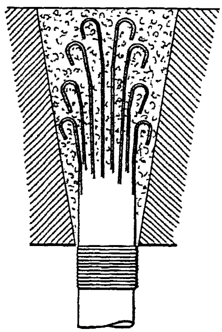


FIG. 61. SECTION THROUGH CAP

cleaned, and tinned. Next, the ends of the wires are bent over and, using a conical mould, are encased in an alloy of low melting point. (Figs. 60 and 61.) An alloy of 83 per cent lead and 17 per cent antimony is sometimes used. Whatever metal is employed, it is important not to overheat the wires. Split conical grips are used to hold the rope in the testing machine. The foregoing method is tedious, and to obtain a successful test care must be exercised in making the cap. A suitable alloy is now specified by the B.S.I.

Long, lightly serrated wedges are sometimes used. They

a spring. This is clearly seen in Fig. 75. The tube is pierced diametrically and carries a fine wire *W* which is viewed by the aid of the microscope *T*. The readings are observed on the micrometer scale in the eyepiece of the microscope. The micrometer screw has 50 threads to the inch and one complete turn corresponds to 50 divisions on the eyepiece scale. When the test piece elongates the displacement of the wire is twice that of the gauge length on account of the leverage about the fulcrum *E*. As a result one division on the scale is equivalent to 0·0002 in. extension on the gauge length. Readings to 0·00002 in. are possible. The standard instruments are for gauge lengths of 8 in. and 2 in.

A modified form is made for measurement of the elastic compression of short blocks on a 2 in. gauge length. Readings can be taken by estimation to 0·002 mm. or 0·000008 in. A clamping bar is provided for registering the clamps on the test piece.

Mirror Extensometers. Mirror extensometers operate on the following principle.

Consider a scale, telescope and mirror set up as indicated in Fig. 76, with the mirror in the position *aa*. Let *oo′* be the line

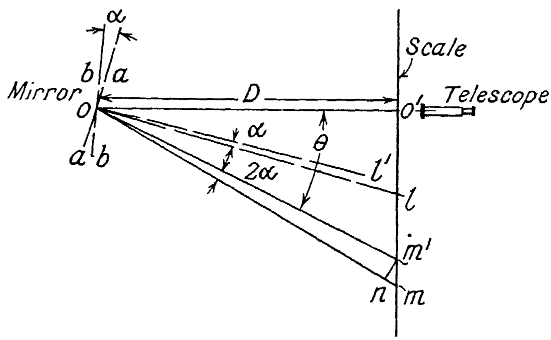


FIG. 76. PRINCIPLE OF MIRROR EXTENSOMETER

of sight and *ol* the normal to the mirror. The scale division seen in coincidence with the cross wire of the telescope is *m*, say, the angle of incidence *mol* being equal to the angle of reflection *loo′*. If the mirror be tilted to the position *bb*, the normal will be *ol′* and the scale division in coincidence with the cross wire of the telescope is *m′* where $\angle\, m'ol' = \angle\, l'oo'$. If α is the angle through which the mirror is tilted this will also be the

angle between the normals, i.e. the angle lol'. It is easily seen that the angle mom' between the two positions of the incident ray is equal to (twice $\angle loo'$ − twice $\angle l'oo'$) $= 2\alpha$.

$$\text{Now} \quad mm' = o'm - o'm'$$
$$= D(\tan \angle moo' - \tan \angle m'oo')$$
$$= D(1 + \tan \angle moo' \times \tan \angle m'oo') \tan (\angle moo' - \angle m'oo')$$
$$= D(1 + \tan \angle moo' \times \tan \angle m'oo') \tan \angle mom'.$$

where D is the distance between the scale and the mirror.

If the angle moo' is small the angle 2α will also be small, and we can neglect the product of the tangents and in place of tan 2α we may write $\tan 2\alpha = 2 \tan \alpha = 2 \sin \alpha = 2\alpha$.

If the angle moo' is of the order of 5°, tan $\angle moo'$ is about 0·1 and the product of the tangents is about 0·01, so the error introduced by this approximation is about 1 per cent.

If mm' corresponds to 1 scale division we have $\alpha = 1/2D$ as the value of the angle of tilt in radians corresponding to one scale division. The distance D between the scale and the mirror must be measured in the same units as the divisions of the scale.

When, as is sometimes the case, a circular scale of radius D is used the foregoing approximation becomes exact.

Another approximation which is sometimes useful when the working angle increases beyond that in which the tangent can be taken as the angle itself is the following—

From the figure, we have

$$\tfrac{1}{2} \angle m'om = \tfrac{1}{2}(nm'/om') = \tfrac{1}{2}mm'(\cos \theta/D\sec \theta)$$
$$= \cos^2 \theta/2D$$

as the value of one scale division in radians.

Bauschinger's Extensometer. Bauschinger introduced a type of instrument in which two clips were pressed against the specimen by springs pressing outwards against caoutchouc rollers. The rollers were carried by a clip and themselves carried the mirrors. Extension of the specimen caused rotation of the mirrors and the rotations were measured by means of scales and telescopes. Results were measurable to 0·0001 mm. A modern form employing steel rollers will be described later.

Morrow's Extensometer. Morrow's instrument is shown in Fig. 77. Two rings R, R, are clamped to the specimen by set

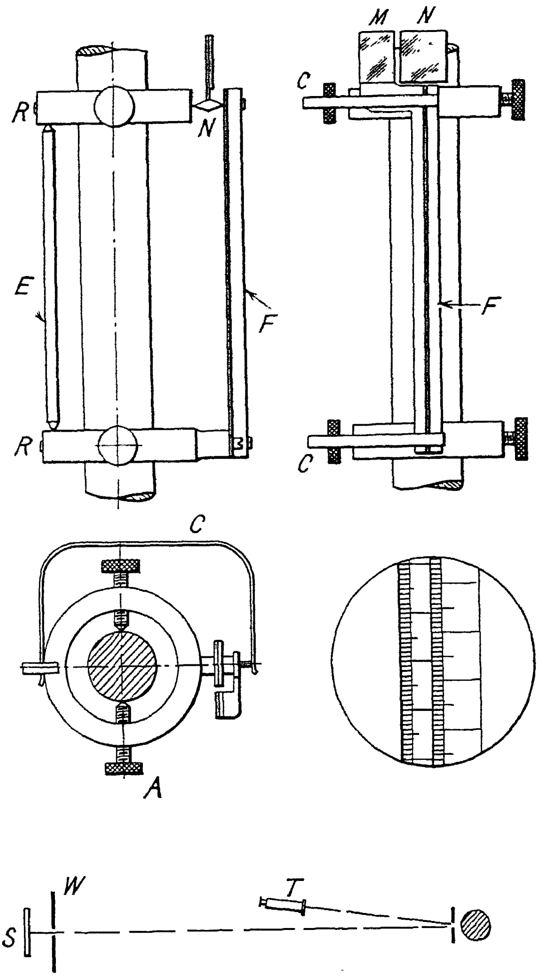


FIG. 77. MORROW'S EXTENSOMETER

screws A round which they pivot. The rod E allows the rings, in addition, to pivot about its ends.

An arm F rigidly attached to the lower ring carries a fixed mirror M at its upper end and supports a rhomb lever which

carries the mirror N. The apparatus is held in position by clips C.

A scale S placed parallel with the axis of the test piece is illuminated by a lamp, not shown, and the reflected beam allowed to pass through an aperture in the screen W to the extensometer mirrors and thence by reflection to the telescope T.

The extension of the specimen causes the rings to pivot round the ends of the rod E, thus inducing relative motion between the upper ring and the arm F. This tilts the rhomb and its mirror and the angle of tilt is observed through the telescope.

The appearance of the scale in the telescope is then as shown in the middle right-hand view in the figure, the image of the scale shown by the tilting mirror appearing displaced from that shown by the fixed mirror.

With a scale divided into fortieths of an inch and with a scale set 80 in. from the mirror, the magnification is 3 000 : 1 so that the smallest reading is

$$\frac{1}{40} \times \frac{1}{10} \times \frac{1}{3\,000} = \frac{1}{1\,200\,000}$$

of an inch.

Lamb's Roller Extensometer. The roller extensometer, Fig. 78, consists essentially of two elements held together against opposite sides of the test piece by means of light springs. Each element is complete in itself and consists of a pair of hardened steel plates separated by two small rollers which are held in position by the springs. Each plate has a sharp knife-edge which bears on the face of the test piece. As the test piece elongates the plates move relatively to each other and give the roller a small rotary movement proportional to the extension of the specimen.

Each roller carries a mirror at one end and at the other a small knurled head for making the final adjustment. No jig for marking the specimen is needed. A pair of dowel pins or a gauge is used to set each element to length before it is clamped to the specimen, the dowels being withdrawn before straining the test piece. The theory of the instrument, as given by Professor Lamb, is as follows—

Referring to Fig. 79, d_1 and d_2 are two rollers—one on each side of the test piece—each carrying a mirror. Let D be the

distance between the scale and mirror and a the distance between the two mirrors. Let θ and ϕ be the incidence and

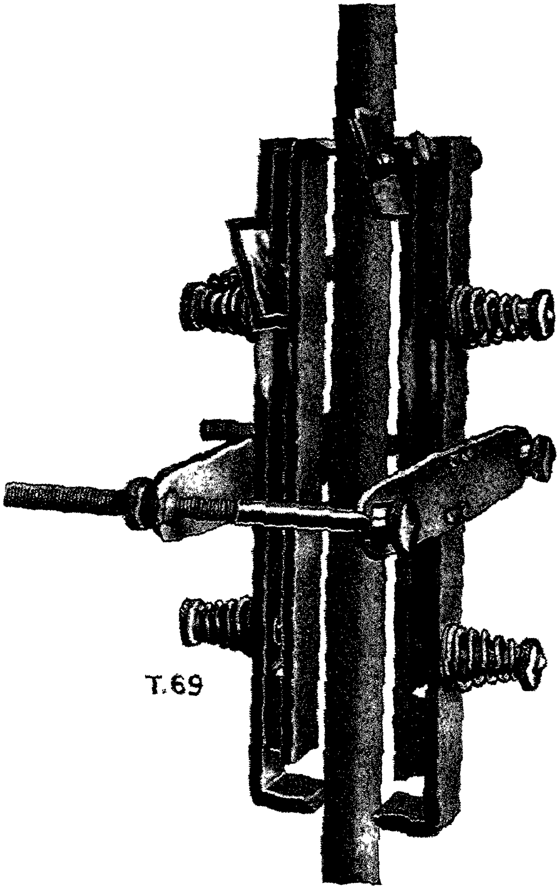


FIG. 78. LAMB'S ROLLER EXTENSOMETER
(*A. Macklow Smith*)

reflection angles at the mirrors A and B and let s be the reading on the scale.

Then, for small opposite rotations $\delta\theta$ and $\delta\phi$ of the mirrors the increment of scale reading is

$$\begin{aligned}\delta s &= 2D(\delta\theta + \delta\phi) + (2a\delta\theta/\cos\phi)\cos\phi \\ &= 2D(\delta\theta + \delta\phi) + 2a\delta\theta \quad . \quad . \quad . \quad . \quad (1)\end{aligned}$$

Again, if e_1 and e_2 are deformations given to each extensometer element, d_1 and d_2 the corresponding roller diameters, we have the relations

$$\left.\begin{aligned} e_1 &= d_1\delta\theta \\ e_2 &= d_2\delta\phi \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad (2)$$

and the axial deformation is

$$e = \tfrac{1}{2}(e_1 + e_2) = \tfrac{1}{2}(d_1\delta\theta + d_2\delta\phi) \quad . \quad . \quad . \quad (3)$$

Now any effect of non-axial loading of the specimen will be

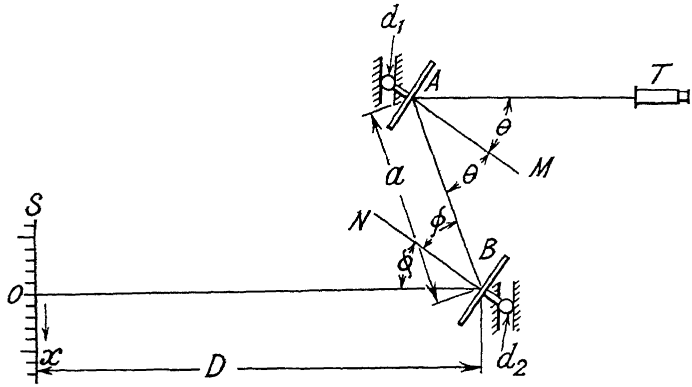


FIG. 79. PRINCIPLE OF LAMB'S EXTENSOMETER

to make a small proportional difference in the values of e_1 and e_2 and we can therefore write

$$e_2 = (1 + k)e_1 = (1 + k)d_1\delta\theta \quad . \quad . \quad . \quad (4)$$

and if there is a small difference in the roller diameters we can (writing d in place of d_1) put

$$d_2 = (1 + \lambda)d \quad . \quad . \quad . \quad . \quad (5)$$

Hence, from equations (2), (4) and (5)

$$(1 + \lambda)\delta\phi = (1 + k)\delta\theta \quad . \quad . \quad . \quad (6)$$

From (1) and (6),

$$\delta s = \left[2D\,\frac{2 + k + \lambda}{1 + \lambda} + 2\alpha\right]\delta\theta \quad . \quad . \quad (7)$$

and from (3), (5) and (6)

$$e = \tfrac{1}{2}d(2 + k)\delta\theta \quad . \quad . \quad . \quad . \quad (8)$$

Combining (7) and (8)

$$e = \frac{d\left(1 + \frac{k}{2}\right)\delta s}{2D\,\frac{2 + k + \lambda}{1 + \lambda} + 2a} \quad . \quad . \quad . \quad . \qquad (9)$$

which, omitting squares and products of small quantities reduces to

$$e = \frac{d(1 + \lambda/2)\delta s}{4(D + a/2)} \quad . \quad . \quad . \quad . \qquad (10)$$

In this result it is to be noted that $d(1 + \lambda/2)$ is the mean of the diameters of the two rollers.

The measurements upon which the accuracy of the performance of the instrument depends are—

(1) diameter of the rollers,
(2) gauge length of the instrument,
(3) the scale distance,
(4) distance between mirrors,
(5) the reading of the scale.

Of these (1) and (2) constitute the permanent calibration of the instrument, the degree of accuracy to which the former is known being of the order of 0·005 per cent to 0·02 per cent according to the size of roller employed, while the latter is within 1 per cent of the nominal length. The magnitude of the errors arising from (3), (4) and (5) depends upon the operator's choice of a suitable distance for the scale in relation to the amount of extension measured. With reasonable care (3) and (4) can be measured easily to an accuracy of 0·2 per cent for any distance of the scale from the instrument and as it is usual to arrange on the average for scale deflections of the order of 30 cm., there is no difficulty in securing an accuracy in the case of (5) of 0·15 per cent. It will be seen, therefore, that the worst possible combination of errors enumerated cannot lead to an error in the result exceeding 0·5 per cent and with the ordinary precautions usual in work of this nature an accuracy of 0·2 per cent is easily obtainable.

Instead of using a scale and telescope, a projector lamp, Fig. 80, may be employed. The projector is set up as near to the mirror as possible, the mirrors are adjusted and the lamp focused until the crosswires are clearly defined. Observations

may be plotted directly on squared paper on a rotating drum.

When a telescope is used it is convenient to employ a millimeter scale at a distance of 5 000 $(d/4)$, where d is the mean diameter of the rollers. In this case 1 cm. represents 0·0002 in. At such distances it is easy to read to $\frac{1}{2}$ mm., or an extension of

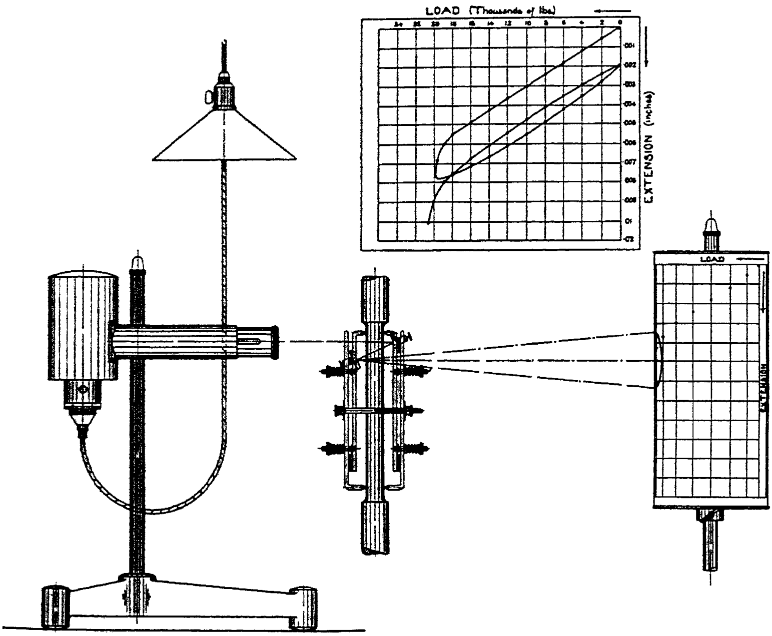


FIG. 80. METHOD OF PLOTTING LOAD-EXTENSION CURVE WITH THE AID OF A PROJECTOR LAMP
(*A. Macklow Smith*)

0·00001 in. Lamb's Extensometer is made by Mr. A. Macklow Smith, Westminster.

The Amsler Mirror Extensometer. Amsler's mirror extensometer is based on the Marten's system and is shown in principle in Fig. 81. In this instrument comparison is made of the amount of the extension of the test piece relative to a standard length in the form of a strip. The *comparison strip*, as it is termed, has at one end, a sharp edge which is placed in one of the gauge marks on the specimen. At the other end of the strip is a notch which carries a little lever supported in the other

gauge mark. The lever, in the form of a rhomb, has sharp parallel edges and carries a small mirror. When the test piece undergoes alteration in length the rhomb is tilted. An elastic clip holds the comparison strip against the specimen and prevents the point of contact of the small lever from being displaced on the test piece without, however, interfering with the angular movement of the lever. The change of gauge length is

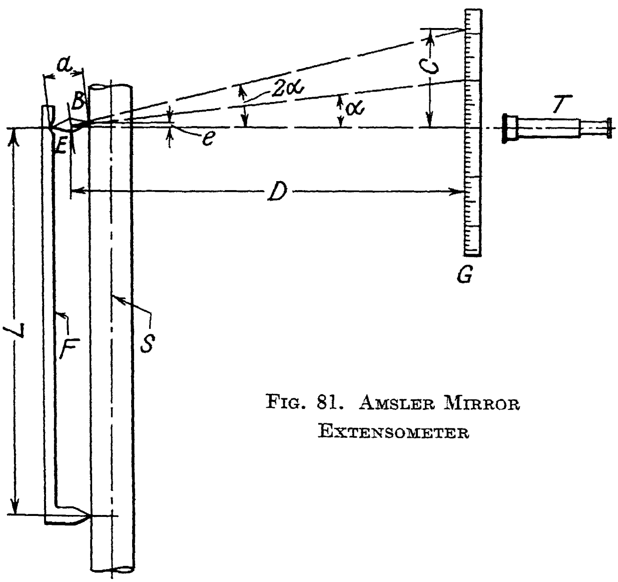


FIG. 81. AMSLER MIRROR EXTENSOMETER

measured by noting the angular movement of the mirror by means of a telescope and scale. Referring to Fig. 81, T is the telescope, B the mirror, E the rhomb or prism lever, S the test piece, F the comparison strip, and G the scale. A similar assembly is arranged symmetrically on the opposite side of the test bar. The illustration shows only one set of measuring parts, and for greater clearness the clip is not shown.

For reasons of stability it is usually preferable to place the edge of the strip F at upper end of the specimen and the mirror carrier at the bottom.

In the diagram, a represents the width of the prism, D the distance between scale and mirror, L the length of the comparison strip measured from the knife-edge to the middle of the

With the previous notation, if S is the sum of the two readings then

$$e/S = a/4D$$

The position of the knife-edge rhomb should be as nearly perpendicular as possible to the surface of the test bar. To facilitate the adjustment of the mirror carrier to the perpendicular position each carrier is provided with an index at right

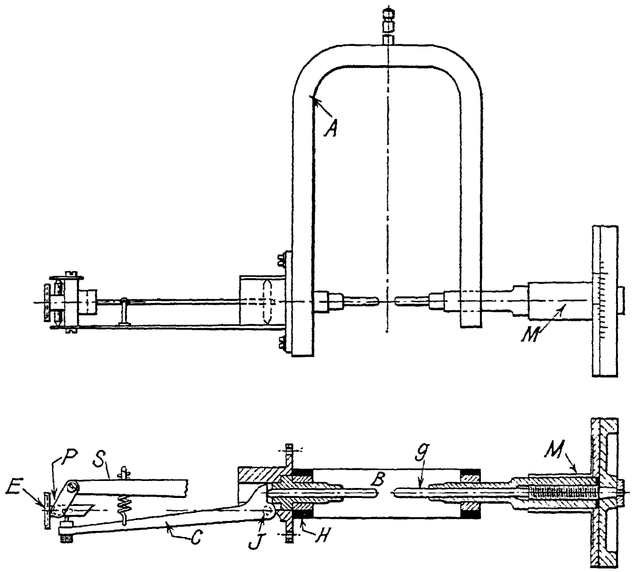


FIG. 82. COKER'S LATERAL EXTENSOMETER

angles to the knife-edge rhomb; its position ought therefore to be parallel to the surface of the test bar for the correct position of the knife.

If desired, a mirror may be attached to the comparison strip alongside the mirror carried by the rhomb. This enables the zero on the scale to be checked as the test proceeds and any bodily shift of the test bar to be allowed for if necessary.

Extensometer for Measuring Lateral Strains. Coker's lateral extensometer is shown in Fig. 82. The U-shaped frame A carries a micrometer M in a friction-tight grip and is provided with a loose measuring needle. A second similar needle B, in line with the first is supported in a casing H, and its outer

end bears against the short arm of a bell-crank lever C, pivoted at J. The longer arm of the bell crank is kept in contact by a light spring with the projecting arm P of the mirror pivoted to a member S of the frame A. Any displacement between the tips of the measuring needles causes a tilt of the mirror and serves to measure the lateral strain produced in a stressed bar placed between them.

The extensometer frame is held in a convenient support when

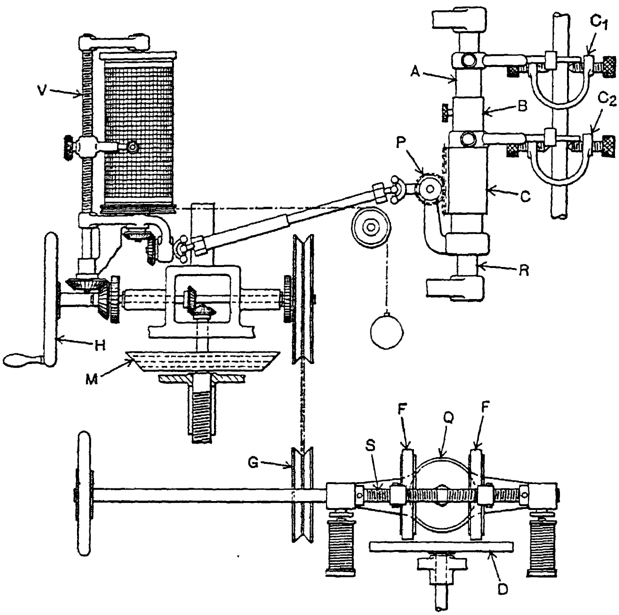


FIG. 83. RIEHLE AUTOGRAPHIC RECORDING GEAR
(*Machinery*)

applied to measure the change of dimension of a specimen. The mirror is employed in conjunction with a lamp or telescope and scale in the usual way.

Autographic Recorders. The Riehle recording gear is shown in Fig. 83. The machine itself is a multi-lever machine similar to the Olsen machine which with its autographic gear was described in Chapter III.

The diagram shows the autographic and automatic gear, the specimen, and part of the poise screw, all rotated into the same plane in order to make the mode of operation clear.

A micrometer dial M is fixed to the poise screw to indicate small increments of load. A bevel pinion on the spindle operating the poise screw transmits motion to the vertical screw V and operates the nut which carries the pencil. The movement of the pencil along the drum is proportional to the movement of the poise weight and provides the load ordinate.

The drum is rotated in the following manner.

The bracket carrying the rod R is bolted to one of the columns on the platen of the machine. A telescopic slide moves on the rod R. The slide consists of three parts: an inner tube A, which slides on the rod; an intermediate tube B which slides on the first; and an outer tube C, provided with friction rollers, free to slide on the tube B. Each can be clamped in position by means of a thumb screw.

The tubes A and C each carry an adjustable fork which rests on a clip pinched on to the specimen at a gauge point. Under operating conditions the inner tube A is allowed to slide on the vertical rod by virtue of the weight of the parts and is supported by the fork resting on the upper clip C_1. The intermediate tube is clamped to A by its screw, while the outer part C is left free on B and supported by the fork resting on the lower clip C_2 on the test bar.

Any movement between the top grip and the upper gauge point results in the tubes moving bodily down the rod with no relative movement. Extension of the specimen results in the tube C moving relatively to A by an amount equal to the extension between the gauge points. The tube C carries a rack which drives the pinion P and this rotates the chart drum through a double Hooke's joint. The rotation of the drum provides the strain ordinate. The magnification is 5: 1.

Automatic operation is obtained by means of a friction drive. The disc D is driven at a suitable speed by a light leather belt not shown. Two fibre discs F, F are secured by a feather to a spindle working in bearings in the swivel plate Q which is supported by the frame of the machine. The swivel is free to oscillate about the vertical axis so as to permit one or other of the fibre wheels to engage the driving disc. When either engages, the spindle is rotated and operates the poise screw in the appropriate direction. This is accomplished by the belt drive from the pulley G. The swivel carries two armatures or keepers, and is controlled by electromagnets, one of which is put in circuit whenever the beam makes contact with the top or

bottom stop. The rate of movement of the poise can be regulated by hand by means of the screwed spindle S. This carries two forks which bear on the fibre discs and can be moved radially across the face of the driving disc with consequent alteration of the velocity ratio. When everything has been set up, the clutch operating the straining gear is thrown in and a

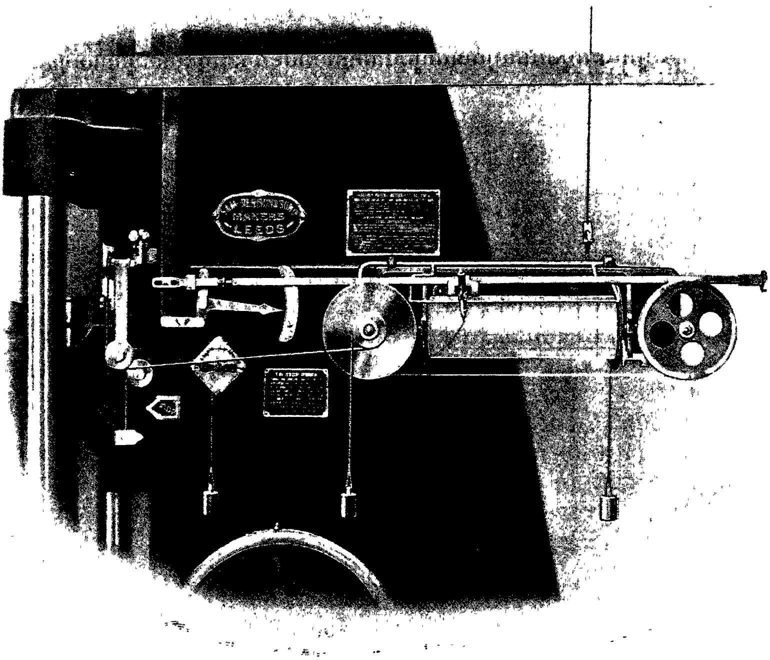


FIG. 84. DENISON AUTOGRAPHIC RECORDER

The steelyard balance indicator is visible to the left and the rod of the compensating gear can be seen behind the right hand end of the drum.

(*Denison & Son Ltd.*)

switch is closed to complete the circuit when the beam touches a contact point. After closing the switch the test proceeds automatically.

The Denison Autographic Recorder, Figs. 84 and 85, is made in two types. The drum carrying the chart is of aluminium and is mounted on a ball bearing spindle. The largest size of graph is 10 in. × 10 in. A stepped pulley having flat grooves for cord driving is mounted on the spindle and provides the choice of

several scales. The pulley drives the drum by friction, a method that facilitates the setting of the pen to zero. The motion of the drum is obtained from the extension of the specimen, clips being attached at the gauge points to take the driving cords. The stress ordinate is provided by the travel of the pen parallel to

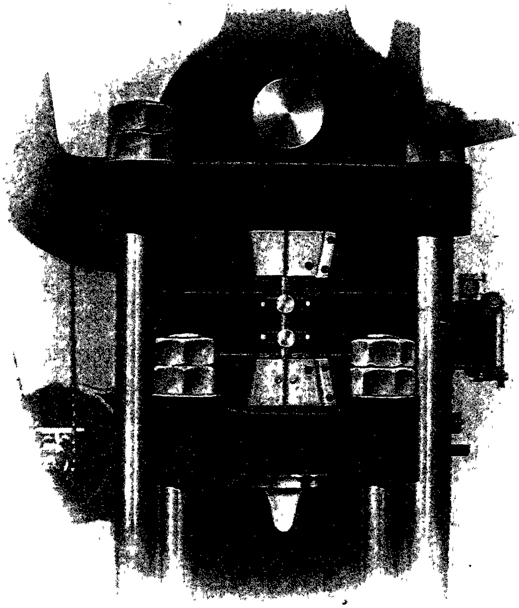


FIG. 85. FRONT VIEW OF DENISON SINGLE-LEVER VERTICAL TESTING MACHINE

With cords in position over specimen frames ready to commence test.
(*Denison & Son Ltd.*)

the drum axis. The pen is carried on a duralumin bar which rests on two broad-faced pulleys mounted on ball bearings. A taut wire attached to the bar passes round the pulleys and ensures that all move in unison. The arrangement is clearly seen in Fig. 85.

The stress ordinate in one type of recorder is obtained from the travel of the poise along the beam. The test is conducted in the ordinary way, the poise weight being traversed to maintain equilibrium.

For single-lever machines a compensating mechanism is fitted which compensates for fluctuations of the steelyard within about $\pm$ 3 per cent of the full capacity of the machine. This is an important feature since it is somewhat difficult to keep the steelyard floating exactly in its mid position. The limits of automatic compensation and the exact position of the steelyard are shown by the indicator.

This type of recorder is the more accurate as it works off the lever and deadweight system of the machine.

In the other type the stress ordinate is produced from the extension of one or more calibrated springs which serve as a load resistant and which automatically maintain equilibrium. Any number of scales may be obtained by the use of appropriate springs. With this type the recorder itself gives the only indication of the test. When fitted to multiple lever machines it is provided with a dashpot for damping out the oscillations of the spring. Means are provided for calibrating the springs.

Other recorders employing a calibrated spring are the Wicksteed and the Buckton-Wicksteed in which the principle is the same as in the instrument just described.

Amsler Recording Extensometer. Amsler's recording extensometer used in conjunction with the Pendulum Dynamometer (page 55) is shown in Fig. 86.

The instrument consists of two knife-edged grips which are held together by two telescopic tubes fitting into each other. Each knife-edged grip is provided with a sharpened disc, with two notches, to bear on the test bar at the gauge point. The round part of the disc is used for attaching the instrument to flat specimens while the notches are used for gripping round bars.

A set of telescopic tubes allows gauge lengths of 8 in., 6 in. and 4 in. to be employed.

When the specimen extends the elongation is transmitted to the recording drum by an inextensible cord. The end of the cord is fixed to an eyelet mounted on one of the grips of the extensometer, the eyelet being kept in position by a spring. The cord next passes round a guide pulley mounted on the other grip and from there is led to the recording drum for the diagram.

The eyelet to which the cord is attached prior to the test gives the cord an initial tension which is effectively maintained as the test bar is stretched.

The grip carrying the guide pulley should always be attached to the test bar end nearest that gripping head of the machine which does not move during the test.

The tubes forming the connection between the two grips are provided with a scale and an indicator showing the reading at every moment of the deformation undergone by the bar. The extensometer is unaffected by the shock when the test piece breaks.

When using plain paper for the diagram, as advocated by Messrs. Amsler, it is advisable, just before starting a test, to mark the zero and maximum lines by holding the pointer on the dial, first at zero and then at the maximum of the load range, and in each position revolving the recording drum with the free hand. The distance between these two basic lines (4 in.) then represents the full capacity of the corresponding range and any in-

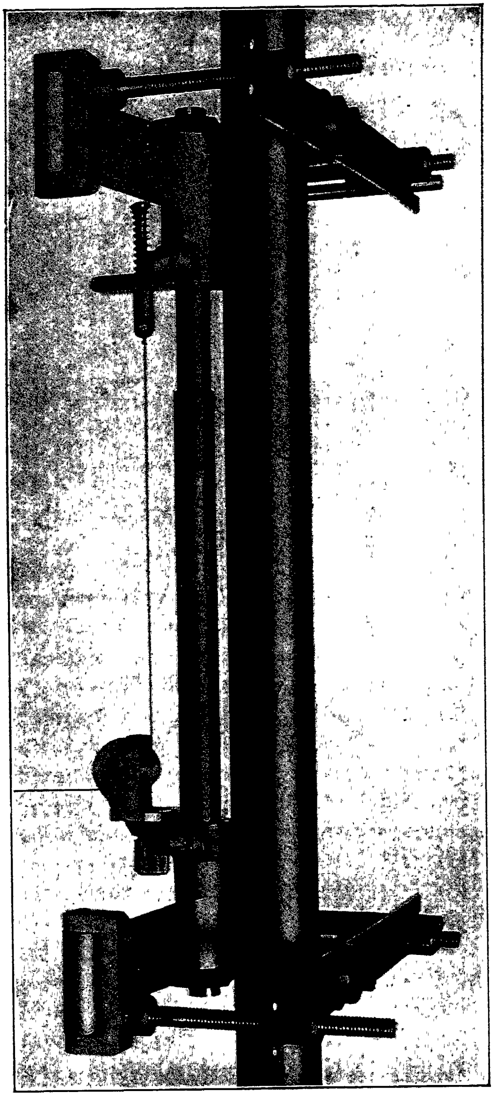


FIG. 86. AMSLER RECORDING EXTENSOMETER
(Alfred J. Amsler & Co.)

termediate position of the diagram can be found by using a draughtsman's scale.

A diagram taken on an Amsler machine is reproduced in Fig. 87.

Messrs. Amsler have developed a new type of recorder, magnifying fifty times and thus giving an enlarged scale for the elastic portion of the stress strain diagram. This is accomplished by means of a micrometer screw with a large head supported by the outer guide tube. The screw is pointed and touches a stop attached to the inner guide tube. One end of a thread wound round the head of the micrometer is led over a fixed guide roller and is loaded with a small weight—the driving weight. The other end of the thread runs over guide rollers on the extensometer and over the pulley on the recording drum. This also is loaded with a weight—the tension weight.

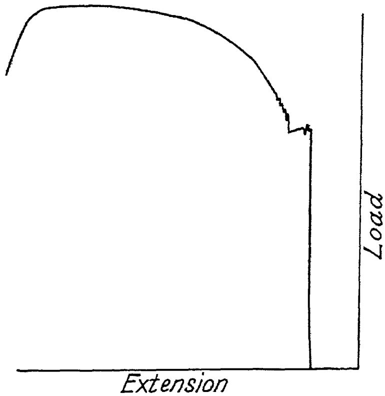


FIG. 87. DIAGRAM TAKEN ON AMSLER MACHINE
(*Alfred J. Amsler & Co.*)

During the elongation of the test bar the upward movement of the upper clamp sets the micrometer screw out of contact with the stop attached to the lower clamp. The micrometer screw, which is thereby set free, revolves under the action of the driving weight, which is heavier than the tension weight, and screws itself down until the point again touches the stop. These movements are transmitted to the recording drum by means of a thread.

By leading the thread over the large pulley on the recording drum the magnification can be increased to 100 times. The graph obtained in this way permits the modulus of elasticity, the limit of proportionality, and the yield-point to be ascertained.

As such a large magnification would be useless beyond the yield point an arrangement to limit the play is provided. This automatically changes the magnifying extension into an ordinary recording extensometer, the elongations being recorded

full size if the thread is led over the large pulley, or double size if it is led over the small pulley on the arm.

This automatic change-over takes place as soon as the elongation of the test bar has reached a dimension corresponding to the yield point which may be expected.

A suspension arrangement prevents the instrument from falling when the test piece fractures. The instrument, which must be used in a vertical position, can be adapted to round bars from $\frac{1}{16}$ in. to 1 in. diameter.

The Westinghouse Extensometer. The Westinghouse extensometer for wires is shown in Fig. 88. It consists of two clips C_1 and C_2 fastened at one end to a block B provided with a slot for the test wire to pass through. The free ends of the clips carry hardened steel rollers R_1, R_2 mounted on pivots. The rollers carry plane mirrors and a spot of light from a lamp in the tube T is projected on to one of the mirrors and passes by reflection to the scale S. Alterations in the length of the test wire of 0·00002 in. can be measured.

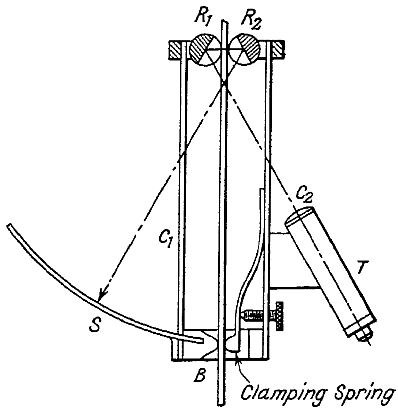


FIG. 88. WESTINGHOUSE EXTENSOMETER FOR WIRES

Dalby's Optical Recorder. Designed to obviate inertia errors, this instrument is shown in Fig. 89. The load measurement is obtained from the extension of a steel bar, the bar acting as a stiff spring. The bar W is hollow and is gripped at its upper end in the head of the machine. A small concave mirror M is free to move about the axis ab, the tilt being effected through the medium of the steel tube T.

The beam of light from the lamp Z is reflected by the mirror Q on to the mirror M and by a third mirror N to the photographic plate at P where the beam is focused to move horizontally across the plate. The magnification is of the order of 340.

One axis of the mirror N is connected to the specimen S by the linkage shown. This consists of a frame F carrying two points which are clamped to the lower gauge point

CHAPTER VII

TORSION TESTING MACHINES

Torsion tests are carried out to determine the modulus of rigidity of a material—the ratio of shear stress to shear strain—or to ascertain its ultimate torsional strength.

Tests are made, most commonly, on specimens of round section. This is the only section to which the elementary theory of torsion is applicable.

The torque T which will induce a stress f in a bar of diameter d is

$$T = (\pi/16)d^3f \quad . \quad . \quad . \quad \text{(see page 5)}$$

When a solid bar of mild steel is tested in torsion it is found that the yield point is less marked than in the tension test. Failure is masked owing to the fact that the whole of the material is not at once stressed to the same degree. The stress is greatest in the outer layers and becomes more and more uniformly distributed over the section as the plastic stage is traversed. There is no appreciable reduction in the cross-section of the test piece and hence the stress-strain curve does not show the pronounced droop at the end as in the tension test, but becomes almost parallel to the strain axis. The shear stress being then uniformly distributed, the maximum stress is calculated by the formula

$$T = (\pi/12)d^3f$$

where T is the breaking torque.

In making a test, it will be found that after the true elastic stage has been passed, the steelyard, for each increment of load, tends to fall due to creep, and that only approximately accurate readings can be taken. Equal intervals of time should be allowed between each successive pair of readings.

In order to determine the yield stress more accurately the test piece is sometimes made in the form of a tube. The stress distribution across the comparatively thin wall of the tube being more nearly uniform, the yielding will be more definite. Thin-walled tubes are not, however, very suitable for the determination of the maximum strength as they frequently deform after creep has begun.

The modulus of rigidity G can be found from the formula

$$G = 32Tl/\pi d^4\theta \quad . \quad . \quad . \quad \text{(see page 6)}$$

when the gauge length l, the diameter d of the specimen, and the angle of twist θ produced by the applied torque T are known.

In the case of tubes of outside diameter D and inside diameter d, the foregoing expressions become respectively

$$T = \frac{\pi}{16}\frac{(D^4 - d^4)}{D}f$$

and

$$G = \frac{32\,Tl}{\pi(D^4 - d^4)\theta}.$$

Ductile materials, such as mild steel, fracture on a plane at right angles to the axis of the test piece, but brittle materials fracture along a 45° helix, indicating that failure occurs where the tensile stress is a maximum.

The usual method of applying a torque to the specimen is by means of a worm and wheel driving a chuck in which one end of the test piece is gripped. The other end of the test piece is held, either in the steelyard directly, or in one arm of a multiple-lever system. The applied torque is balanced by the moment of the poise about the fulcrum of the weighing lever.

While many universal testing machines have provision for making torsion tests it is often advantageous to employ a separate machine for this purpose. A good example is the machine made by Messrs. W. & T. Avery Ltd., which permits tests to be made in reverse torsion.

The machine is illustrated in Fig. 90. The specimen is gripped in chucks *A* and *B*. Torque is applied by turning the handwheel *H* which drives through the worm gear *F*. The floating sleeve carrying the chuck *A* is free to slide longitudinally and so prevents an end load from being imposed on the test piece as twisting proceeds.

The torque is transmitted by the chuck *B* to the cross-lever *L* fitted with knife-edges *E* at its ends.

According to the direction of the twist, the main lever pulls on one of the intermediate levers which in turn communicates the pull to the steelyard. The latter carries the balance weight *D* and the movable poise *w* operated by the small handwheel *J*. The main lever has a fulcrum consisting of hardened steel cones and cups bearing upon rings of hard steel balls. The

connection between the handwheel, which is on a fixed portion of the frame, and the gearing on the steel yard is made at the point of no motion, that is, at the fulcrum; consequently, the pressure upon the handwheel is not communicated to the steelyard. This obviates the possibility of the specimen being broken prematurely. The steelyard is fitted with a polished scale plate graduated from zero to 2 000 lb.-in. by 50 lb.-in. divisions:

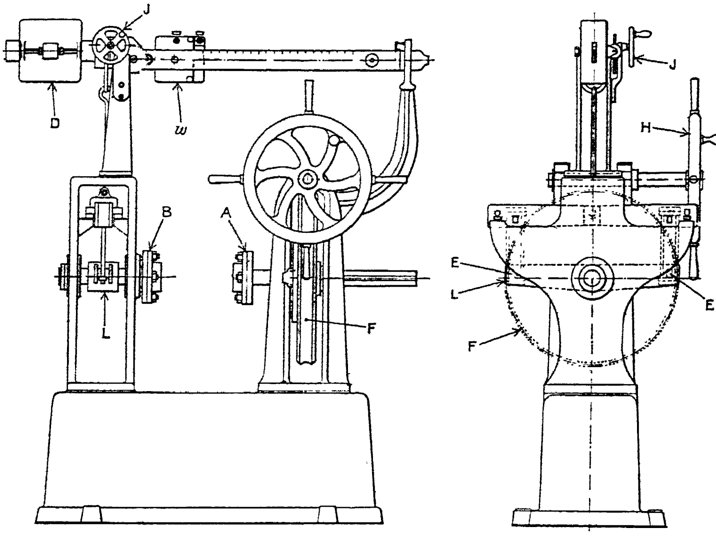


FIG. 90. AVERY TORSION TESTING MACHINE
(*Machinery*)

a vernier scale on the poise subdivides these into divisions of 5 lb.-in.

The full capacity of the machine is 10 000 lb.-in. and additional poise weights are supplied for this capacity. When the poises are combined the scale readings need to be multiplied by 5.

A graduated ring mounted on the wormwheel enables angles of twist on the whole length of specimen to be measured over the plastic stage. The specimen used in this machine has coned ends each provided with two keyways, as shown in Fig. 91. The maximum length of specimen between shoulders is about 10 in.

As this type of specimen is somewhat expensive to make, it is an advantage, for ordinary work, to have additional holders to take specimens having square ends.

For measurements within the elastic stage some form of torsion strain meter is needed. Several commercial instruments are available for making such measurements.

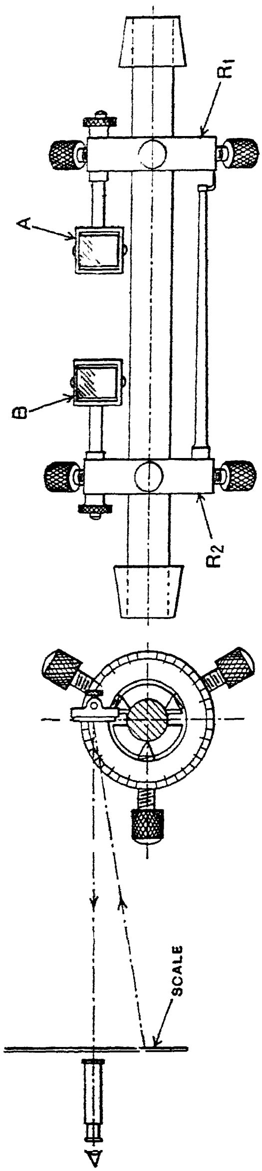


FIG. 91. SIMPLE FORM OF MIRROR TORSION METER (*Machinery*)

A simple and effective arrangement by which the strains can be observed is shown in Fig. 91. Two rings, R_1 and R_2, each carrying a mirror (A, B) which can be tilted about two axes at right angles, are clamped to the gauge points. These, used in conjunction with two scales and two telescopes, enable observations of the angular displacement to be made. If the two mirrors are arranged close together it is possible to work with only one telescope and one scale.

Measurements of the twist during the plastic stage may be made if one ring is provided with a scale of degrees and the other with a suitable pointer the end of which traverses the scale on the first ring.

The table on page 119 gives the results of a test on a mild steel specimen $\frac{7}{8}$ in. diameter and 6 in. between gauge points.

Referring to Fig. 91, suppose the mirror A to be adjacent to the end of the test piece to which the torque is applied, while the other, B, is near the end gripped by the main lever. In the table, column 3 contains the observations taken on mirror A and column 5 those taken on mirror B. If the initial reading, namely 16, of column 3 be subtracted from the remaining numbers in the column

TABLE VII

RECORD OF TEST OF A MILD STEEL BAR $\frac{7}{8}$ IN. DIAMETER

No. of Reading (1)	Applied Torque T lb. in. (2)	Observed Reading Mirror A (3)	Corrected for Zero (4)	Observed Reading Mirror B (5)	Corrected for Zero (6)	Twist on 6-in. Gauge Length (7)
1	—	16·0	—	10·3	—	—
2	850	26·6	10·6	14·7	4·4	6·2
3	1 100	28·6	12·6	15·5	5·2	7·4
4	1 225	29·9	13·9	15·7	5·4	8·5
5	1 350	31·25	15·25	15·8	5·5	9·75
6	1 425	31·5	15·5	15·9	5·6	9·9
7	1 475	31·9	15·9	15·8	5·5	10·4
8	1 600	33·1	17·1	16·2	5·9	11·2
9	1 720	34·2	18·2	16·6	6·3	11·9
10	1 820	35·4	19·4	16·8	6·5	12·9
11	1 975	36·5	20·5	17·25	6·95	13·55
12	2 100	37·7	21·7	17·25	6·95	14·75
13	2 350	39·8	23·8	17·7	7·4	16·4
14	2 600	41·7	25·7	17·65	7·35	18·35
15	2 850	44·3	28·3	18·5	8·2	20·1
16	3 100	46·2	30·2	18·6	8·3	21·9
17	3 150	56·7	40·7	19·7	9·4	31·3

and tabulated in column 4, the values so obtained represent the twist up to the ring R_2. The differences between corresponding numbers in columns 4 and 6 give the values of the twist, in scale divisions, on the given gauge length. These values are tabulated in column 7.

The distance between the mirror and the scale being denoted by L, the angle of twist θ radians is found from the formula

$$\tan \theta = \frac{\text{twist in scale divisions}}{2\,\text{L}}.$$

In most cases it is sufficiently accurate to replace tan θ by θ itself. For an angle of 5° the difference between the radian measure of the angle and its tangent is, from four-figure tables, 0·0002 and for an angle of 10° the difference is 0·0018.

In the present instance the scale readings are in centimetres and the distance between the scale and mirror is 12 ft. 6 in. Hence the multiplying factor for converting the readings in column 7 into radian measure is

$$\frac{1}{2 \times 12{\cdot}5 \times 12 \times 2{\cdot}54} = 0{\cdot}001313.$$

If, in the setting up of the test piece, a small initial torque be inadvertently applied, the correct value of the torque can be ascertained at the end of the test by plotting the torque-twist diagram and producing the straight portion of the graph to intersect the vertical axis through the origin. The magnitude

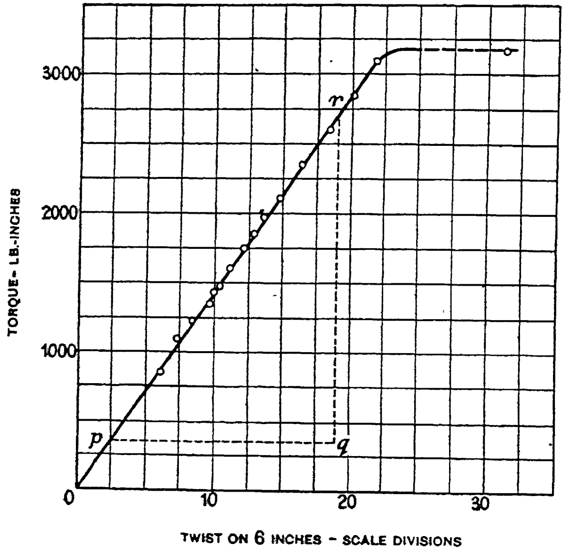


FIG. 92. TORQUE-TWIST DIAGRAM FROM TEST OF $\frac{7}{8}$ IN. DIAMETER MILD STEEL BAR
(*Machinery*)

of the intercept will give the value of the initial torque. The diagram can then be replotted, if desired, with the correct zero.

The results in columns 2 and 7 are plotted in Fig. 92.

To determine the modulus of rigidity we have

$$\begin{aligned} G &= 32Tl/\pi d^4\theta \\ &= (32l/\pi d^4) \times (T/\theta) \end{aligned}$$

The ratio T/θ is given by the slope of the graph. Thus

$$G = \frac{32l}{\pi d^4} \cdot \frac{T}{\theta}$$

$$= \frac{32 \times 6}{3{\cdot}1416 \times (0{\cdot}875)^4} \cdot \frac{qr}{0{\cdot}001313pq}$$

$$= \frac{32 \times 6}{3{\cdot}1416 \times (0{\cdot}875)^4} \cdot \frac{2\,350 \text{ lb.-in.}}{0{\cdot}001313 \times 16{\cdot}5 \text{ scale divisions}}$$
$$= 11\,300\,000 \text{ lb. per in.}^2$$

Fig. 93 shows the results of a torsion test on a mild steel bar pierced transversely.

In the torsion machine Fig. 94, made by Messrs. Alfred J. Amsler & Co., of Schaffhouse, the load is measured through the

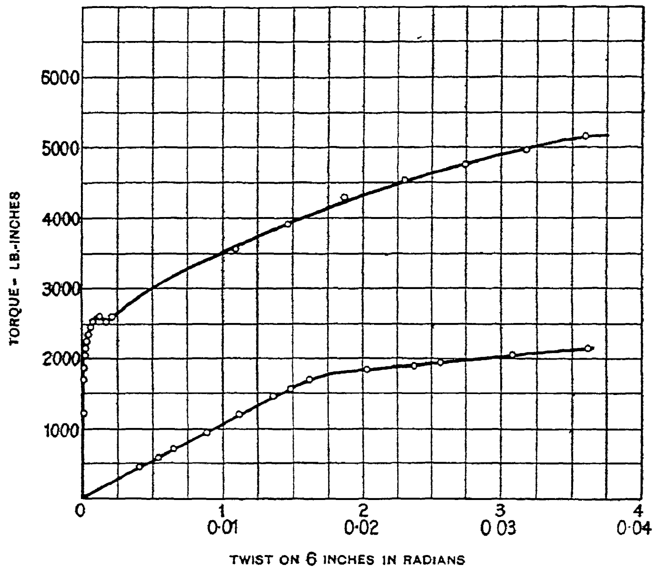


FIG. 93. TORQUE-TWIST DIAGRAM OF TEST OF $\frac{7}{8}$ IN. DIAMETER MILD STEEL BAR PIERCED TRANSVERSELY
(*Machinery*)

medium of a pendulum. The gripping head at the left of the machine is fixed to the end of a shaft which rotates in ball bearings and is rigidly connected to the pendulum rod. When the test bar is in position the torque applied to the worm drive tends to rotate the pendulum, which is then deviated from its position of equilibrium to such an extent that its static moment balances the applied torque.

To prevent the pendulum from falling back too quickly at the instant when the test bar fractures, it is hooked on to a brake rope which prevents a too rapid descent. The inclination of the pendulum is transmitted to the spindle of a pointer which indicates on a dial the moment of torsion in lb.-ft.

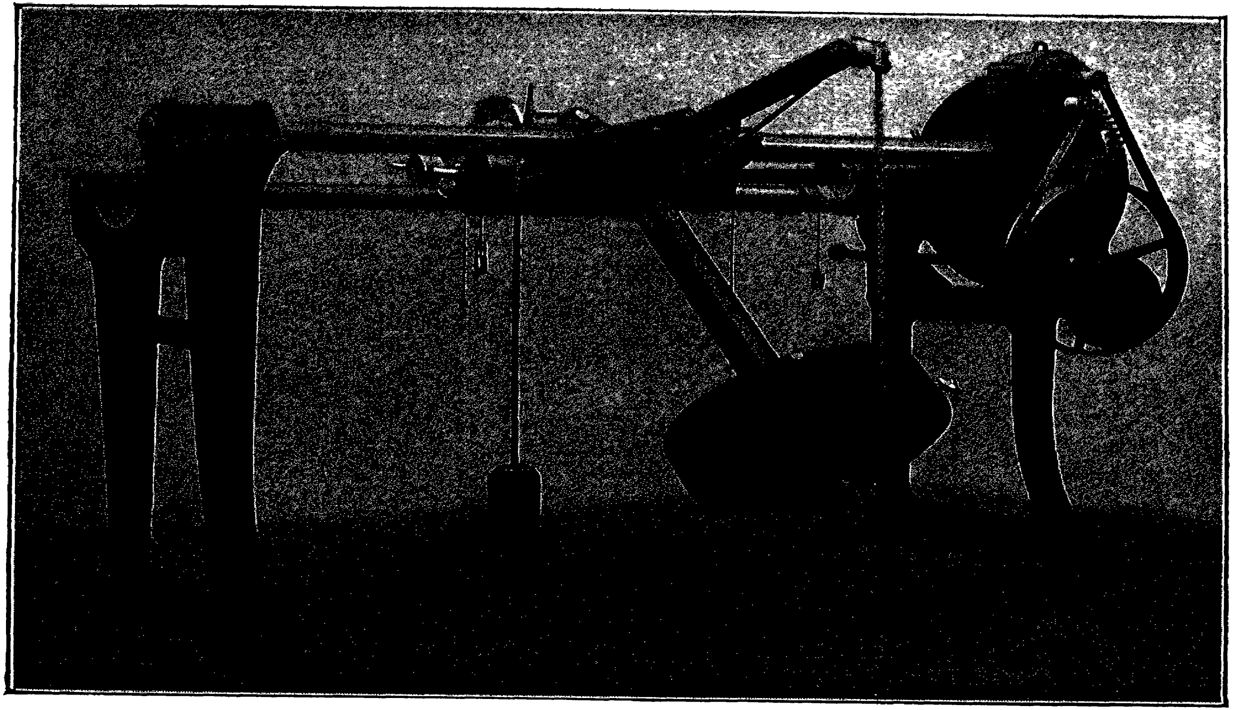


FIG. 94. AMSLER PENDULUM TYPE TORSION TESTING MACHINE
(*Alfred J. Amsler & Co.*)

A record of the test is given by an autographic device similar in principle to that described in Chapter VI.

The deflection couple of the pendulum can be changed by altering the position of the bob and a separate dial is provided for each degree of sensitivity produced.

For gripping the test bar, taper-wedge grips are provided, smooth wedges being used for flat specimens and toothed wedges for round bars. In making torsion tests on smooth

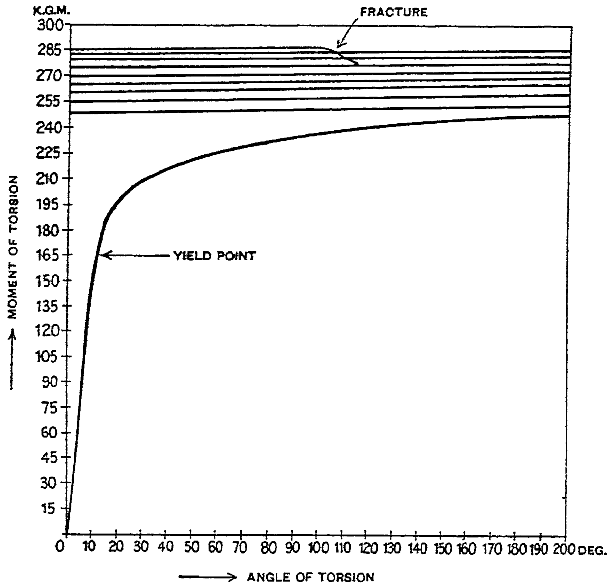


FIG. 95. AUTOGRAPHIC RECORD TAKEN ON AMSLER MACHINE
(*Machinery*)

round bars the ends are enclosed with wood slats which are gripped in the toothed wedges. When testing tubes it is necessary to fit plugs in the ends in order to prevent the tubes from being flattened at the gripping point. Special gripping heads are provided for dealing with crankshafts when it is required to test these in torsion.

The twist on a given gauge length is measured by means of two graduated rings and two discs moving inside the rings. The rings are clamped to the horizontal bars of the machine and the discs to the specimen at the gauge points. The angle of torsion can be read to $\frac{1}{5}°$.

indicated on a counter actuated from the drive of the lower clamping head. Angles of twist may be measured to one-hundredth of a revolution.

The angle of torsion of the measuring wire is shown on a circular scale mounted on the intermediate arm. A driving pin which is inserted in the spindle and to which is secured the lower clamping head of the wire under test, pushes a loose pointer over the circular scale. The distance traversed by this pointer depends on the torsional resistance of the measuring wire, and in consequence is a measure of the torsional moment applied to the test wire.

The angle of torsion of the measuring wire as a function of the angle of torsion is determined by experiment.

The device for calibrating the measuring wire consists of weights which exert equal and opposite forces on the ends of a cross-pin fixed to the spindle of the lower head of the measuring wire. For calibrating very thin measuring wires the checking weights are suspended according to the bifilar principle. The calibrating arrangement, which is only needed occasionally, is removed from the apparatus during torsion tests.

Four measuring wires are provided of 50, 5, 0·5 and 0·05 lb.-ft. capacity.

The free length of the test specimens between clamping heads is 4 in. or 2 in., according to the settings of the upper and middle arms. Wires from the thinnest up to $\frac{9}{64}$ in. diameter can be gripped. The illustration shows the machine arranged for motor driving. In this case the lower clamping head rotates at about 20 r.p.m.

CHAPTER VIII

HARDNESS TESTING

Hardness. An important property of a material is its hardness, a property easily comprehended in a general way, but one that eludes precise definition and whose existence as a definable quality is doubted.

The usual interpretation placed upon the term hardness is that of resistance to penetration, or resistance to abrasion. But these views are quite distinct since a material may offer considerable resistance to abrasion and yet be relatively soft according to standards based on indentation tests.

Hardness tests, as usually made, involve the breakdown of the material tested. The result is that the distribution of stress is not known with certainty and hardness values are therefore based on the consequences of the stresses produced.

Although such tests are merely relative they are of great value to engineers. Primarily, hardness, however it may be understood, is important in that it enables a material to withstand certain conditions of service. Its importance in the scheme of mechanical testing lies in the fact that its quantitative determination affords an estimate of the tensile strength of steel and wrought iron and, moreover, throws light on the treatment which the material has received during manufacture. The ease with which hardness tests can be made has led to their widespread adoption in commercial testing.

Scratch Tests. The mineralogical scale of hardness is based on a scratch test and consists of a number of substances arranged in an empirical series. The arrangement indicates that each substance will scratch the one preceding it in the scale but not the one that succeeds it. The scale, usually termed *Moh's Scale*, is as follows—

1. Talc.	6. Felspar.
2. Gypsum.	7. Quartz.
3. Calcspar.	8. Topaz.
4. Fluorspar.	9. Corundum.
5. Apatite.	10. Diamond.

Here there is no attempt at measurement, the relationship being purely qualitative.

The scratch test has, however, been developed by Martens and other workers and tests have been introduced in which a diamond, loaded by a movable poise on a lever, scratches the test piece. Hardness is then defined as the load in grams under which a conical diamond would produce a scratch 0·01 mm. wide. In recent years further attention has been given to "scratch hardness" but this aspect of hardness testing will not here be pursued further. Instead, attention will be devoted to the indentation test, as this is now standardized and plays an important part in the workshop testing of metals.

Indentation Tests. Indentation tests are made under static conditions and consist in forcing a ball or pointed body into the test piece under a dead load. Conical and pyramidal points, as well as spheres, are used. The former offer some advantage, but the general run of hardness tests are, at present, made with a ball.

In the *Brinell method*, from which the test takes its name, a steel ball is forced into the test piece under pressure and the hardness of the material is expressed as a number—the Brinell hardness number—which is defined as the stress per unit of spherical area.

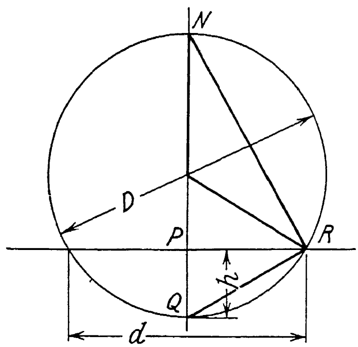


Fig. 97. Illustrating Method of Deriving the Brinell Hardness Number

If P is the load applied in kilogrammes and A the spherical area in square millimetres, the hardness number H, is given by

$$H = P/A = P/\pi D h$$

where D is the diameter of the indenting ball and h the depth of the indentation produced, measured as indicated in Fig. 97.

If d is the diameter of the impression we have from the similar triangles NPR, RPQ, in the figure,

$$PQ/PR = PR/NP$$

or

$$\frac{h}{d/2} = \frac{d/2}{D - h}$$

from which $h^2 - hD - d^2/4 = 0$

Consequently $$h = \frac{D \pm \sqrt{(D^2 - d^2)}}{2}$$

and for the depth of the impression to be less than half the diameter of the ball the minus sign must be taken, giving

$$h = \tfrac{1}{2}[D - \sqrt{(D^2 - d^2)}].$$

By a rule of mensuration the area A of the spherical surface is πDh, that is

$$A = \pi(D/2)[D - \sqrt{(D^2 - d^2)}]$$

and the hardness number calculated on the diameter of impression as a basis, is

$$H = \frac{P}{(\pi D/2)[D - \sqrt{(D^2 - d^2)}]}.$$

In cases where it is necessary to make the numerical calculation the expression may be put into a more convenient form thus—

$$(D/2)[D - \sqrt{(D^2 - d^2)}] = (D^2/2)[1 - (1 - d^2/D^2)^{\frac{1}{2}}]$$

$$= \frac{D^2}{2}\left[1 - \left(1 - \frac{1}{2}\frac{d^2}{D^2} - \frac{1}{2}\cdot\frac{1}{4}\left(\frac{d^2}{D^2}\right)^2 - \frac{1\,.\,1\,.\,3}{2\,.\,4\,.\,6}\left(\frac{d^2}{D^2}\right)^3 \ldots\right)\right]$$

$$= \frac{d^2}{4}\left(1 + \frac{d^2}{4D^2}\right)$$

neglecting the sixth and higher powers of d/D.

So that

$$H = \frac{P}{\frac{\pi d^2}{4}\left(1 + \frac{1}{4}\cdot\frac{d^2}{D^2}\right)}.$$

The error involved by the use of this approximation is less than 2 per cent even when the ratio $d/D = 0{\cdot}6$, the largest value advised in ordinary testing.

It is customary, whenever possible, to employ a load of 3 000 kg. and a ball 10 mm. diameter. The diameter of the impression is measured by means of a microscope. In some forms of instrument the microscope carries a cross wire and is moved bodily over the field of view by a micrometer screw. The cross wire is first brought into coincidence with one edge of the impression and the reading of the micrometer dial noted. The cross wire is then brought into coincidence with the opposite

edge of the impression and the reading again noted. The difference between the two readings gives the diameter of the indentation.

In other instruments the image of the impression is focused on a scale in the eyepiece whereby the diameter can be read off directly.

Two measurements of the indentation should be taken at right angles to each other and the mean of the two values used in calculating the hardness number.

For commercial testing the microscope should be capable of measuring the diameter of the impression to 0·05 mm.

The surface of the test piece should be smoothly finished and if a small ball of 1 or 2 mm. diameter is being used the surface of the specimen should be brought to a polish. No. 0 emery cloth is satisfactory for loads of 30 kg. or more. For very small loads the finish should be with No. 00 or 000.

The British Standard Specification requires that the centre of the impression shall be not less than two and a half times the diameter of the impression from any edge of the test piece, and that the thickness of the test piece shall be such that no marking showing the effect of the load shall appear on the underside.

The value of the hardness number is affected by the diameter of the ball, by the pressure, and by the distortion of the ball itself. On the latter account it is recommended that with hardness numbers above 500 care should be taken to see that the balls are considerably harder than the material tested.

Experiments by Meyer showed that the mean pressure per unit area was constant when the indentations were geometrically similar; that is, $4\ P/\pi d^2 =$ a constant.

It follows that as d/D and $4P/\pi d^2$ are constant, for similar indentations on the same material, P/D^2 is also constant. The importance of this relation is that it enables comparative tests to be made where a 10 mm. ball and a load of 3 000 kg. are not applicable.

Thus, if with a small test piece, a ball of diameter D_1 is used, the corresponding load P_1 that should be applied is—

$$\text{For a 5 mm. ball } P_1 = 30 \times 5^2 = 750 \text{ kg.}$$

$$\text{For a 1 mm. ball } P_1 = 30 \times 1^2 = 30 \text{ kg.}$$

The operation of indenting the material results in a ridge

being formed around the impression, Fig. 98 (*a*), or in a depression as in Fig. 98 (*b*). The former is noticeable in copper and mild steel; the latter in manganese steel and some bronzes.

As the hardness numbers, as calculated from the curved area of impression, are not strictly comparable for different materials, the depth of the indentation below the original surface has been suggested as forming a more rational basis of comparison. This, however, is not the basis of the Brinell hardness number according to the British Standard Specification, which stipulates that the hardness number must be calculated from the diameter and not from the depth of impression.

The hardness numbers calculated from the diameter of the impression are not independent of the load, but if the depth

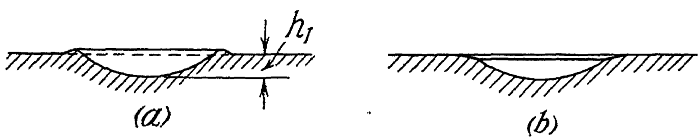


Fig. 98. Test Indentations
(*a*) Showing ridge round the edge of the indentation.
(*b*) Showing depression round the edge of the indentation.

from the original surface h_1 be used, the numbers obtained are, for some materials, independent of the load. Carrington has shown, however, that there are considerable divergences from this rule.

For steel and materials of a like degree of hardness, a load of 3 000 kg. with a ball 10 mm. diameter is employed. This makes the ratio $P/D^2 = 30$ as shown previously. For copper alloys the specified value of P/D^2 is 10, while for copper it is 5 and for lead and tin it is unity.

The effect of time on the size of the impression formed has little effect after the first 10 sec., at least for steels. The minimum time of application of the load given by the specification is 15 sec. when $P/D^2 = 30$, and 30 sec. when $P/D^2 = 10$, 5 or 1. The rate of application of the load may, however, cause appreciable differences in the hardness numbers obtained. To avoid errors of this nature some machines, such as the Herbert power-operated machine, are designed to eliminate the personal effect when applying the load.

In practice the hardness number for a given diameter of ball and of impression is found from Tables. The curve in Fig. 99 shows how the Brinell hardness numbers vary with the diameter of impression.

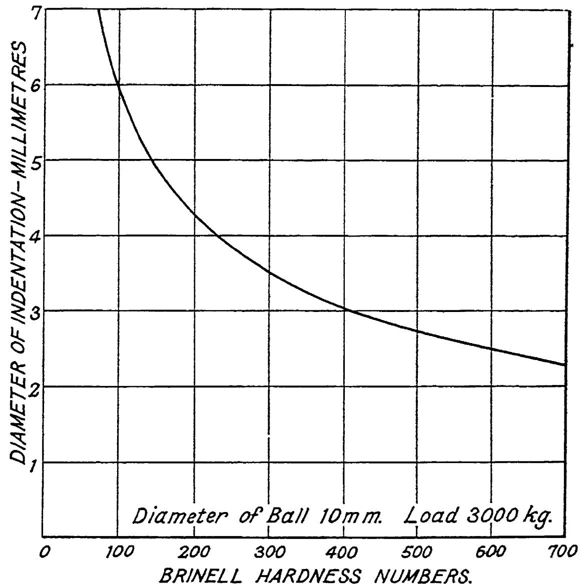


FIG. 99. GRAPH CONNECTING DIAMETER OF INDENTATION WITH HARDNESS NUMBER

Relation Between Brinell Hardness Number and Tensile Strength. The Brinell test is often used to obtain an indication of the strength of a material, since a useful relation exists between the Brinell hardness number and the tensile strength, at least in the case of certain steels. So far, however, no rule of general application has been found to exist.

The chain dotted lines drawn through points represented by circles in Fig. 100 link up the results of a large number of tests made by Messrs. Hadfield and Main on a variety of steels. The British Standard Specification suggests that for steel the tensile strength in tons per square inch can be found approximately by multiplying the Brinell hardness number by 0·22. The full line in the diagram is drawn with this value for its slope. The points marked +, which represent values of

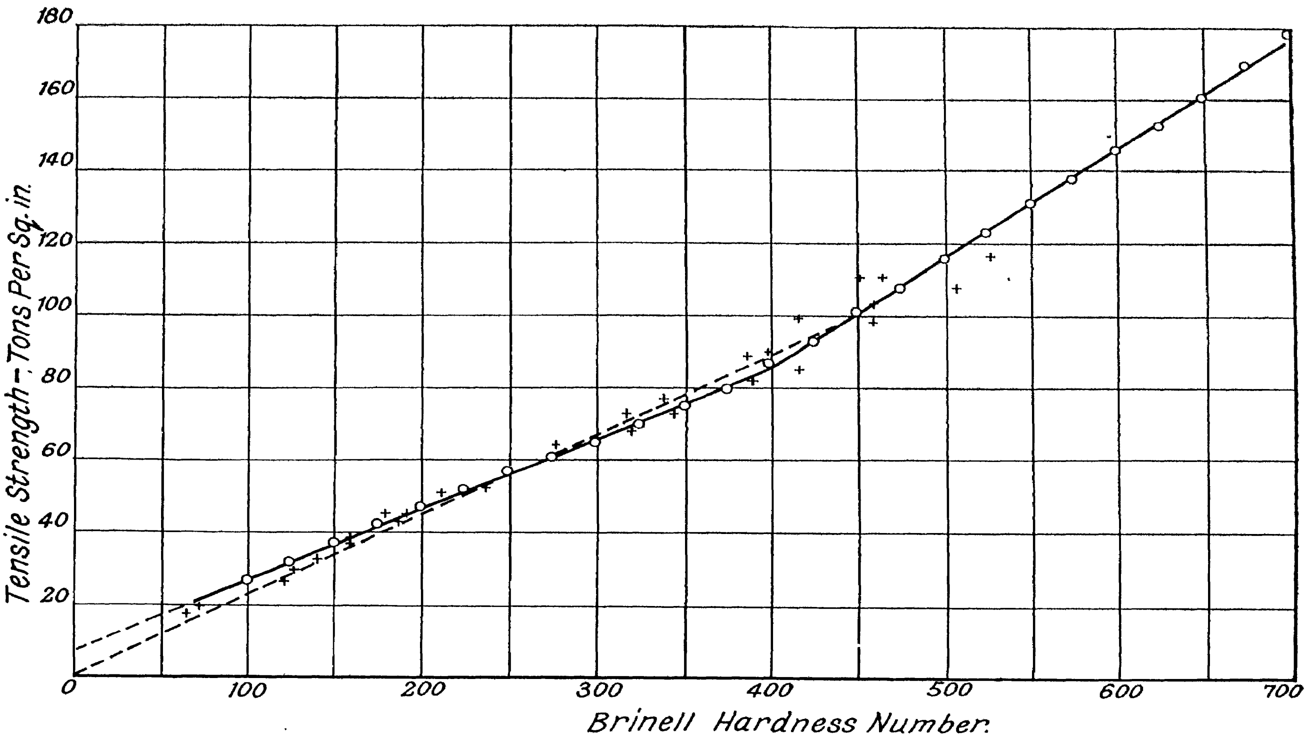


FIG. 100. HARDNESS NUMBER AND TENSILE STRENGTH OF STEEL

the Brinell number, are plotted from various published tests and serve to show how closely the ratio 0·22 holds.

Messrs. Greaves and Jones from an examination of over 1 000 specimens recommend the following values—

Material	Tensile Strength in Tons per in.^{2} = Brinell Number Multiplied by
(a) For heat-treated alloy steels with a Brinell number of 250 to 400	0·21
(b) For heat-treated carbon steels and for alloy steels with a Brinell number below 250	0·215
(c) For medium carbon steels as rolled, normalized or annealed	0·22
(d) For mild steels rolled, normalized or annealed	0·23

The above values do not apply to severely cold worked or austenitic steels.

For non-ferrous wrought alloys such as duralumin and Y-alloy

$$\text{Tensile strength} = \frac{\text{Brinell hardness number}}{4} - 1.$$

Hardness Testing Machines. Numerous machines for making the Brinell test are now on the market. The loading is generally direct or by means of a lever, by screw gearing or by hydraulic pressure.

Messrs. Alfred Herbert Ltd. make a small machine for determining the hardness of specimens of the order of 0·01 in. in thickness. Amongst the applications of this machine may be cited the testing of thin walled tubes without internal support, and of cutlery blades which would be disfigured by a large impression, the determination of the hardness of wire at successive stages of drawing, and the hardness of a cutting tool close to the edge.

The balls used are 1 or 2 mm. diameter and loads up to 50 kg. are employed. For testing extremely soft material balls of 5 mm. diameter are supplied.

Alfred Herbert Small Hardness Tester. The machine, shown in Fig. 101, is the outcome of investigations made at the Research Department at Woolwich Arsenal with the object of evolving a means of determining accurately the hardness of thin specimens. The table for the reception of the work to be

prescribed period the upper handwheel is turned back to take the weight off the specimen, which is then removed and the diameter of the impression measured. For this purpose a microscope is supplied, magnifying 180 diameters and with gradua-

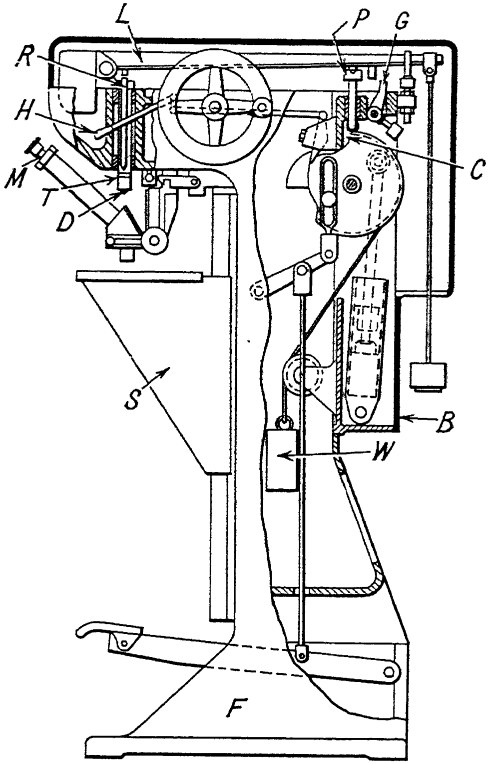


FIG. 102. VICKERS HARDNESS TESTING MACHINE
(*Vickers-Armstrongs Ltd.*)

tions of $\frac{1}{200}$ mm. on the scale. Additional objectives may be obtained giving one-half and one-fifth of the standard magnification respectively.

The Vickers Pyramid Hardness Testing Machine. The Vickers pyramid hardness testing machine, Fig. 102, consists of a frame F of U-section which carries the stage S and a beam L of 20 : 1 ratio. The load is applied through a thrust rod R to

the tube *T* which is free to reciprocate vertically. The tube carries a diamond indenter *D* at its lower end.

Attached to the main frame is a frame *B*. This carries the control mechanism. The plunger *P* is reciprocated by means of the rotating cam *C* and serves to apply and release the test load. The cam is mounted on a drum and when the starting handle *H* has been depressed the whole is rotated by a weight *W* attached by a flexible wire, the speed of rotation being controlled by a piston and oil dashpot.

The rate of displacement of the oil is regulated by an adjustable control valve. The plunger carries a rubber pad at its upper extremity and this engages with a cone mounted in the beam, thereby ensuring a slow and diminishing rate of application for the last portion of the load. The cam both lowers and raises the load, the object aimed at being to eliminate inertia errors. Depression of the foot pedal returns the cam, drum and weight to their original positions.

A trip-piece *G* supports the beam during this latter operation and drops out as soon as the plunger returns to its top position. The machine is then ready for another test and no external power is required other than that provided by the operator in depressing the foot pedal.

The microscope *M* is capable of measuring to 0·001 mm. It is mounted on a hinged bracket so that it can be swung to a position immediately over the impression.

Instead of the usual scale or eyepiece micrometer, a specially designed micrometer ocular is used. The impressions are read to knife-edges, thus avoiding eyestrain, and readings are taken entirely from a digit counter mounted on the side of the ocular. In making a test the pedal should first be depressed in order to load the machine. The area to be tested is placed on the stage and the latter raised until the surface to be tested just clears the point of the diamond indenter. The starting handle is then pressed to release the mechanism, when the test proceeds automatically and terminates with an audible click. Measurements are made across the corners of the square impression in the following manner.

The left-hand knife-edge is adjusted by means of a knurled thumb screw to correspond with the left-hand corner of the impression and the right-hand knife-edge, which is controlled by a micrometer screw connected to the counting mechanism, is moved to correspond with the right-hand corner of the

impression. The view through the microscope then appears as in Fig. 103. The reading is taken from the figures on the counter at the side of the eye-piece and converted by means of tables to Vickers pyramid numerals.

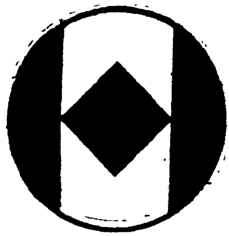
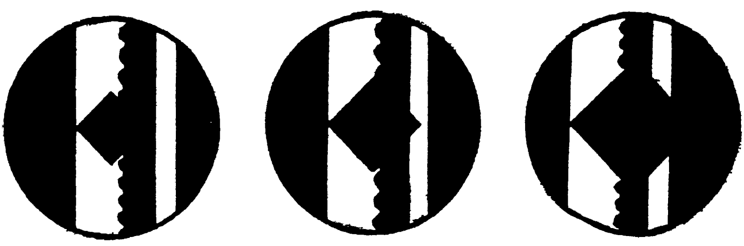


FIG. 103. METHOD OF SETTING THE VICKERS MICROSCOPE
(*Vickers-Armstrongs Ltd.*)

In cases where work has to be tested with a view to ascertaining whether or not it conforms to specified maximum and minimum limits of hardness, a third knife-edge is brought into use. This third knife-edge is brought into the field of vision by turning a thumb screw at the side of the ocular and is set by means of the micrometer right-hand knife-edge to correspond with the smaller dimension, i.e. the maximum limit of hardness. The micrometer knife-edge is then adjusted to correspond with the larger dimension, i.e. the minimum limit of hardness.

Having made the setting of the second and third knife-edges in this manner it is only necessary, when reading, to set the fixed knife-edge to the left-hand corner of the impression in the ordinary way. Fig. 104 (*a*) shows the material too hard,



(*a*) (*b*) (*c*)

FIG. 104. TESTING FOR CORRECT HARDNESS
(*Vickers-Armstrongs Ltd.*)

Fig. 104 (*b*) shows the material to be correct, and Fig. 104 (*c*) shows that the material is too soft.

In handling mass routine work a convenient method of operation is first to make all the impressions on the pieces, using a jig for locating each piece beneath the diamond. When this has been done the readings are made with the microscope, the

pieces being easily set in position by means of the jig. Up to two hundred tests an hour can be made in this way. The machine is manufactured by Messrs. Vickers-Armstrongs Ltd., Crayford, Kent.

The Olsen-Brinell Hardness Tester. In the Olsen-Brinell hardness tester, Fig. 105, the ball is attached to the underside of a piston to which fluid pressure is applied. The test piece is placed on the table and raised by means of the handwheel until it makes contact with the ball. The piston is a ground fit in its cylinder and without packing, so that when pressure is applied by a small hand pump frictional effects are negligible. The pressure is indicated on the gauge, but to ensure correct loading the plunger carrying the weights seen in the illustration fits into the cylinder above the load piston and ensures that an overload is not applied to the ball.

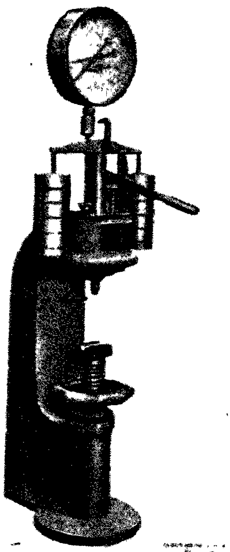


FIG. 105. OLSEN-BRINELL HARDNESS TESTING MACHINE
(Edward G. Herbert Ltd.)

When the maximum pressure is reached the plunger and weights "float" and thus limit the load on the piston. Oil leaking past the piston or plunger is drained away to the reservoir. The pressure is released by opening a small valve on the top of the pressure chamber.

Pyramid Hardness Numerals. The advantage of using the pyramid indenter lies in the similarity of the impressions produced. The hardness numerals obtained by using a pyramidal form are independent of the load. This can be seen from the test results plotted in Fig. 106.

The scale of numerals depends on the angularity of the pyramid. The angle selected as a standard in the Vickers system is 136° which, as may be seen from Fig. 107, agrees with the cone angle for a ball impression of 0·375 times the diameter of the ball. Tables of Diamond Pyramid Hardness Numbers are now issued by the British Standards Association (B.S.S.427); the loads specified being 5, 10, 20, 30, 50, 100, and 120 kg. The

numerals obtained with a pyramid of this angularity are identical with those obtained in the Brinell test under appropriate conditions.

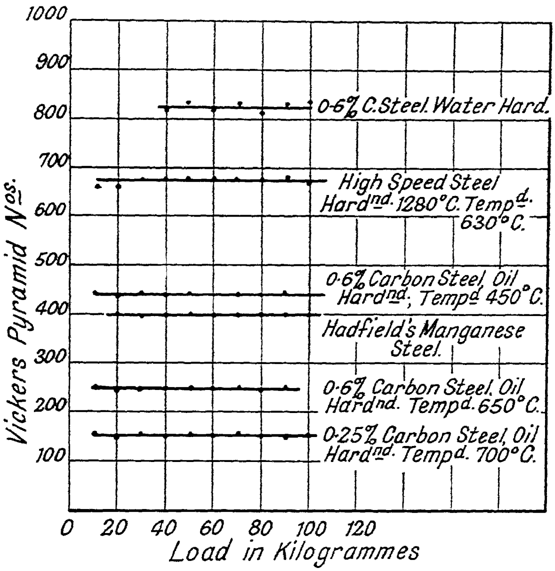


FIG. 106. SHOWING THAT PYRAMID HARDNESS NUMERALS ARE INDEPENDENT OF THE LOAD

The numerals obtained with the Vickers 136° diamond pyramid are termed Vickers pyramid numerals, abbreviated as

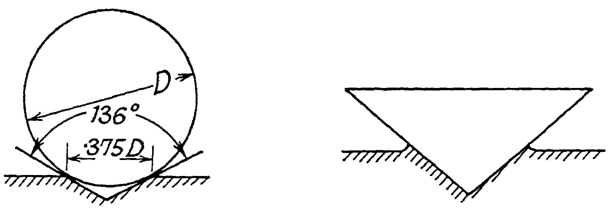


FIG. 107. COMPARISON BETWEEN BALL AND DIAMOND IMPRESSIONS

. . . ° V.P.N. As in the case of the ball the hardness number is defined as the ratio

$$\frac{\text{Load in kg.}}{\text{Surface area of indentation in mm.}^2}$$

If

H = pyramid hardness number.

P = pressure in kg.

d = mean diagonal of impression in mm.

θ = angle between each pair of opposite faces of the pyramid,

then, referring to Fig. 108,

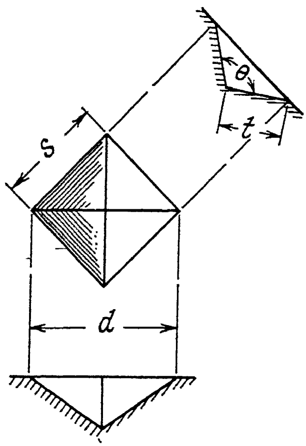


FIG. 108. DIMENSIONS OF DIAMOND IMPRESSION

$$\begin{aligned}\text{Surface area of indentation} &= 4 \times \text{area of one face}\\ &= 4 \times (st/2)\\ &= 4 \times (s/2) \times (s/2)\ \text{cosec}\ (\theta/2)\\ &= (d^2/2)\ \text{cosec}\ (\theta/2);\end{aligned}$$

therefore

$$H = \frac{2P \sin \theta/2}{d^2}$$

For an angle of 136°

$$H = 1{\cdot}854P/d^2$$

The similarity between the two systems of numerals obtains only in the lower regions of hardness—that is, where the steel ball does not undergo any appreciable deformation. Between

500 and 600 Brinell hardness this deformation begins to make itself manifest by yielding slightly lower readings than the diamond pyramid, and this tendency increases with increasing hardness until it becomes very pronounced. The real relationship is illustrated by curve *A* in Fig. 109. Curve *B* in the same

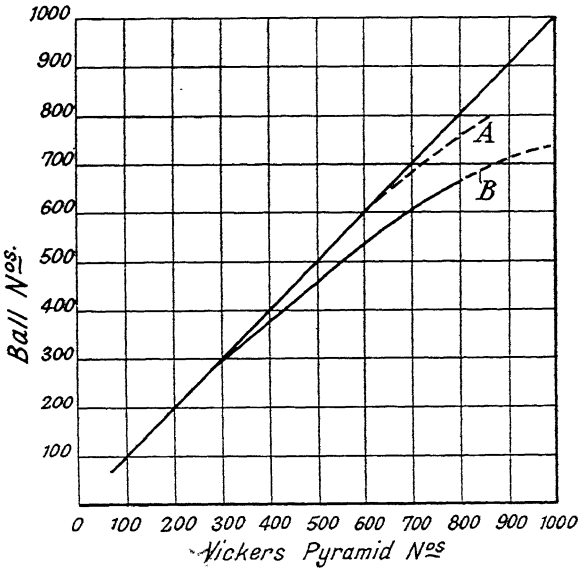


FIG. 109. CURVE SHOWING THE RELATION BETWEEN THE NUMERALS WITH THE VICKERS PYRAMID AND THOSE OBTAINED WITH A STEEL BALL

A loaded to give an impression of which the diameter is equal to 0·375 times the diameter of the ball.
B is a 1-cm. steel ball with a load of 3 000 kg.

figure shows the relation between the hardness numerals obtained with a Vickers pyramid and Brinell numerals obtained in the normal way, that is with 3 000 kg. load and a 10 mm. ball. The disparity is greater here than in the curve A, because in addition to deformation of the ball in the latter case the load is constant. The ball impressions decrease in size with increase in hardness of the material and the hardness numbers are disproportionately low. Unavoidable differences in the hardness of different steel balls will affect the numerals obtained, particularly in the higher regions. On this account the higher portions of the curves are shown dotted.

Rockwell Hardness Tester. Another static hardness tester is the Rockwell machine, Fig. 110, largely used in America. A steel ball $\frac{1}{16}$ in. diameter or a 120° diamond cone with a rounded point is used, and the depth of the indentation is automatically recorded on a dial. The load is applied by the hand screw seen in the figure. To obviate errors caused by the spring of the machine a load of 10 kg. is first applied and the clock indicator

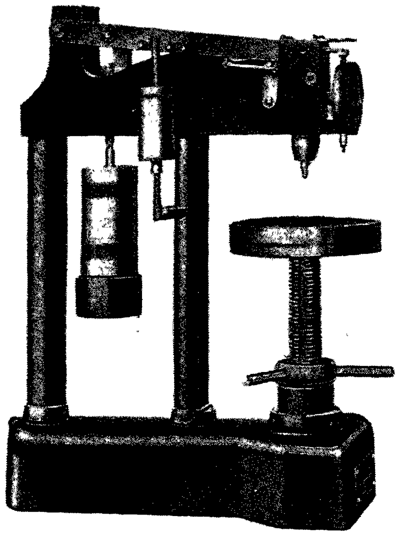


FIG. 110. ROCKWELL HARDNESS TESTER
(George H. Alexander Ltd.)

set to zero. The load is then increased to 100 kg. in the case of the ball and to 150 kg. in the case of the diamond. The load is then taken off except for the initial 10 kg. and the hardness numeral read off from the dial. Two scales are employed; one, the "C" scale, for the diamond and the other, the "B" scale, for the steel ball.

In Fig. 111 curves are given for the approximate conversion of Rockwell's "B" scale to Brinell hardness numbers. The machine is rapid in action and articles may be tested at the rate of 250 per hour.

Shore Scleroscope. A dynamic hardness test is provided by

the Shore scleroscope in which a small pointed tup weighing about 0·0052 lb. is allowed to fall freely from a height of 10 in. on to the test piece.

The height of rebound of the tup is measured against a scale graduated into 140 equal divisions. The result is dependent on

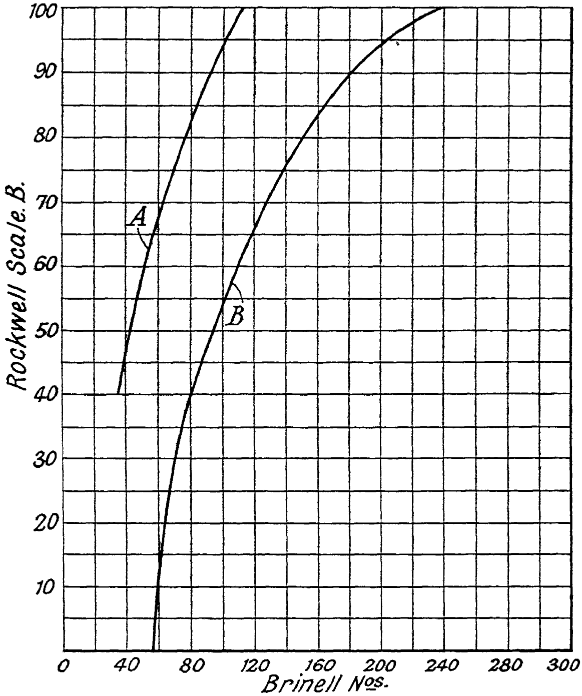


FIG. 111. CONVERSION TO STANDARD BRINELL NUMBERS OF ROCKWELL B. SCALES

(*a*) Rockwell B scale $\frac{1}{16}$ in. ball and 60 kg. load.
(*b*) Rockwell B scale $\frac{1}{16}$ in. ball and 100 kg. load.

(*Institution of Automobile Engineers*)

the permanent deformation produced in the test piece at the point of impact, the rebound being diminished on account of the work of deformation.

As most materials are hardened by cold working it is important when making tests to see that the material is not struck twice in the same spot.

Relations Between the Various Systems of Hardness Numbers. No precise relationship exists between scleroscope, Rockwell

square impressions, Fig. 113. This effect has been examined by O'Neill, who finds that the concavity is due to a raised extruded ridge formed along the faces of the pyramid, while convexity is due to a depression of the edge. Soft copper, soft iron and quenched steel show a slight apparent convexity, while drawn copper and rolled steel appear to give concave indentations. The effect introduces an error into the measurements whereby

FIG. 113. RIDGE EFFECTS IN DIAMOND IMPRESSIONS

worked metals will give high results and very workable metals will give low results.

Herbert Pendulum Hardness Tester. Another apparatus for making dynamical hardness tests is the pendulum hardness tester of Messrs. Edward G. Herbert Ltd. It consists, as its name implies, of a pendulum weighing 4 kg. supported on a ball 1 mm. diameter or on a 1 mm. ball-shaped diamond, the whole constituting a compound pendulum of $\frac{1}{10}$ mm. length. Immediately above the ball is a graduated weight mounted on a screw. By raising or lowering the weight the centre of gravity of the instrument may be brought to a predetermined distance below the centre of the ball. Above the weight is a curved spirit level and a scale reading to 100. The pendulum is shown mounted on an operating stand in Fig. 114.

Six practical tests can be made with the instrument:

(1) A TIME HARDNESS TEST. When the pendulum is allowed to rest on the specimen the ball makes an indentation the size of which depends on the hardness of the specimen. The pendulum is allowed to oscillate through a small arc and the time of swing noted. The time of swing, measured by a stop watch, gives a measure of the hardness.

Microscope readings are not needed. The method of testing eliminates distortion of the ball which occurs when measurements of Brinell hardness above 600 are being made, and the use of a ball-shaped diamond enables tests on steels of 1 000 Brinell hardness to be made.

TABLE VIII

TYPICAL HARDNESS NUMBERS

Using a 1 mm. steel ball

Glass	100
Very hard carbon steel	75
Hard carbon steel	65
Heat-treated h.s. steel	52
Annealed h.s. steel	26
Mild steel	20
Rolled brass	15
Cast brass (soft)	11
Lead	3

The approximate Brinell hardness may be obtained by multiplying diamond time hardness by 13·5 and steel ball time hardness by 10.

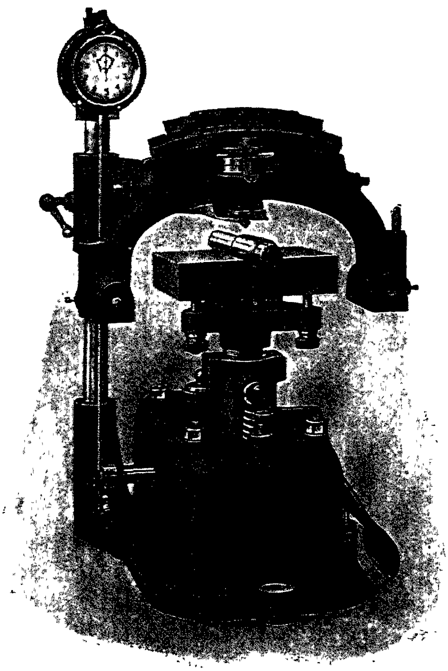


FIG. 114. HERBERT PENDULUM HARDNESS TESTER

(*Edward G. Herbert Ltd.*)

(2) TIME-WORK-HARDENING TESTS. Metals in service become work-hardened in varying degrees, and a knowledge of their work-hardening properties is of importance; as for example in metals for deep drawing or pressing, free cutting steels, and rail, tyre and gear steels, in which work-hardening increases their resistance to wear.

To make the time-work-hardening test the time-hardness test (1) is first made and the specimen is then work-hardened by rolling it with the pendulum. A second time-hardening test is then made. The processes are repeated alternately and continued until the hardness reaches a maximum (the "maximum induced hardness") and then declines.

Some representative results are given below.

TABLE IX

TIME-WORK-HARDENING TESTS

Material	Original Time Hardness	Passes of Ball					
	0	2	4	6	8	10	12
Hard tool steel	71	89	88	86	—	—	—
Manganese steel	21	45·5	52·4	54	56·2	57·2	44·6
Mild steel	21	30	30·8	30·9	31·2	32·3	31·6
Stainless steel	18	38·7	41·9	43·1	43·1	44	43·8
Loco. tyre steel	28·6	35	36·2	36	—	—	—
Deep drawing steel	12	19·4	20·7	20·6	—	—	—

(3) SCALE TEST. If the pendulum be tilted and placed on a specimen with the bubble at O on the scale it will indent the specimen as before. When released it will swing and elongate the impression by rolling, the length of swing indicating the resistance of the materials to rolling.

The readings obtained in this test generally place materials in the same order of hardness as the time test but the "scale-hardness" numbers do not correspond with the time-hardness numbers. The scale test is very sensitive and is chiefly used in detecting changes in hardness, especially those caused by heating or working the metal.

(4) HOT HARDNESS TEST. Specimens can be subjected to a time-hardness test with the specimen in an electric furnace, the temperature being measured by pyrometer. A ball-shaped diamond is necessary in this test as a steel ball would be affected by the high temperature.

(5) TEMPERATURE-WORK-HARDENING TEST. Work-hardening tests on unhardened metals have shown that these usually lose their work-hardening capacities at comparatively low temperatures and regain them at higher temperatures, although the indentation hardness remains nearly constant.

A typical work-hardening curve of Vickers test bar steel is given in Fig. 115. Three peaks, P_1, P_2, P_3 and three depressions D_1, D_2, D_3 are frequently present, the principal depression D_1 occurring at 120° to 140_x C. in steels and below 100° C. in other metals.

(6) SCALE-WORK-HARDENING TEST. If the scale test has been made as described in (3) and at the end of the first swing the pendulum is tilted to 100 and then released, the reading on the reversal will usually be higher, as the specimen has been work-hardened by the ball. If this is repeated from 0 to 100 alternately, the succession of readings obtained will show the progressive increase of hardness due to working.

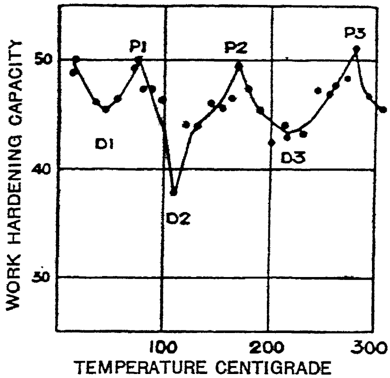


FIG. 115. WORK HARDENING CURVE OBTAINED WITH THE PENDULUM HARDNESS TESTER
(*Edward G. Herbert Ltd.*)

The work-hardening capacity of a metal is measured by the average increase of scale hardness caused by rolling four times with the pendulum ball.

Theory of the Pendulum Hardness Tester. It has been suggested by E. G. Herbert that three simple types of hardness are involved, namely, *plastic indentation hardness* (Brinell hardness); *elastic indentation hardness*—a measure of which is given by the pendulum-time-test on highly elastic materials; and *flow hardness* as measured by the scale-time ratio. Dr. W. J. Walker, however, suggests that only two material properties are involved, *viscous* or *plastic hardness* and *elastic hysteresis hardness*.

Timoshenko has put forward the following simple theory of the instrument.

Let e, Fig. 116, be the distance of the c.g. (G) of the apparatus from the centre O of the ball, W the weight of the pendulum, I its moment

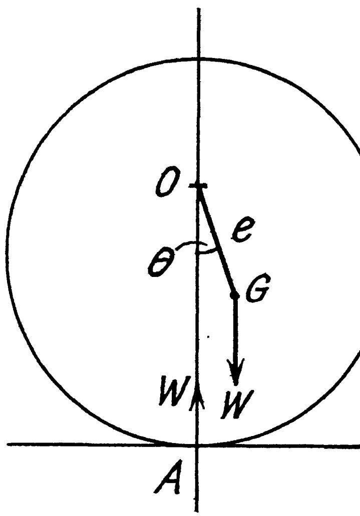


FIG. 116. ILLUSTRATING TIMOSHENKO'S THEORY OF THE PENDULUM HARDNESS TESTER

of inertia about an axis through O perpendicular to the plane of oscillation.

Assuming e and θ to be small quantities the equation of motion when the apparatus is supported on a rigid plane surface is

$$I(d^2\theta/dt^2) + eW\theta = 0$$

from which the periodic time of one swing, that is, one half of a complete oscillation, is

$$T = \pi\sqrt{(I/geW)}.$$

This result is assumed to apply in the case of an elastic plane surface as the distribution of pressure over the area of contact will be symmetrical about the vertical axis.

When permanent set occurs, Fig. 117, the distribution of pressure is no longer symmetrical about the vertical axis OA. Timoshenko assumes the length of the arm of the couple to vary as $(e + \delta)$ where δ is a constant depending on the permanent set, so that the time of one swing is now

$$T = \pi\sqrt{[I/g(e + \delta)W]}$$

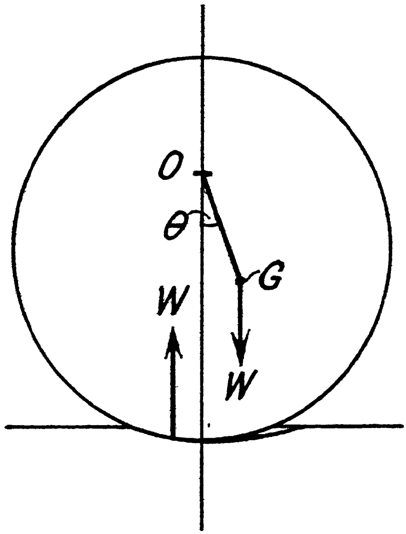


FIG. 117. ILLUSTRATING TIMOSHENKO'S THEORY OF THE PENDULUM HARDNESS TESTER

In a comparison of calculated and experimental results on glass and high-speed steel the latter formula gave consistent results.

The first formula shows the time of oscillation to be inversely proportional to $\sqrt{e}$ and thus independent of the hardness qualities of the material. For some comments on this theory and for some experimental results see *The Engineer*, 1923 (6th April, 6th July, and 7th September).

CHAPTER IX

NOTCHED-BAR IMPACT TESTING

Impact Tests. Static tests, although furnishing valuable information, are nevertheless insufficient to bring out all the characteristics of a material that bear on service conditions. In a great many cases in practice the loading is not of a static character and hence arises the need for a study of the effects produced by dynamic loading.

Of several forms of dynamic test one which has come into prominence in recent years is the *notched-bar impact test* in which a specimen in the form of a notched-bar is tested as a beam under the impact of a hammer or pendulum.

The notch localizes the stress and determines the position of fracture. The test piece may be gripped at one end and tested in the form of a cantilever, or it may be supported at its ends and tested as a beam freely supported.

Considerable divergence of opinion exists as to the meaning and value of the notched-bar test. Research shows that the test is more sensitive the sharper the notch, that the result does not depend greatly on the type of machine used, and that the results, within the limits of ordinary testing, are not unduly influenced by the striking velocity.

The term "impact test" is somewhat misleading as the result indicates not the resistance of the material to shock or impact, but rather differences of condition of the material which cannot be demonstrated by any other test.

The chief value of the test lies in indicating whether or not the heat treatment of a steel has been carried out in a satisfactory manner. It is generally conceded that the condition of a steel giving a high impact value is better than that of the same steel which gives a low impact value. Conclusive evidence on this point appears to be lacking, but the view is supported by the results of experience.

The notched-bar test figures prominently in Air Board Specifications and among the reasons given for its inclusion are—

(*a*) That it is the test which gives most information as to whether the heat treatment of a steel is satisfactory or otherwise.

(*b*) That if the steel has been heat treated to the best advantage the impact value will be relatively high, but if the heat treatment has not been satisfactorily carried out the impact value will be relatively low.

The impact value, however, must not be judged by itself but must be considered after taking account of the type of steel upon which the result was obtained.

Of machines for making impact tests the best known are the

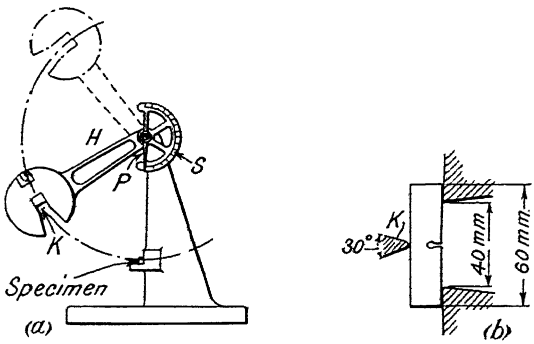


FIG. 118. CHARPY IMPACT TESTING MACHINE
(*a*) Principle of machine. (*b*) Setting of test piece.

Frémont, Charpy, Izod, and Guillery, the last being a rotary-disc machine.

Particulars of these are given in the following table.

The Charpy Machine. The Principle of the Charpy machine is illustrated in Fig. 118 (*a*). The hammer or pendulum *H* swings on ball bearings and is in the form of a disc provided with a vertical knife *K*. The specimen being simply supported as shown at (*b*), Fig. 118, the pendulum is raised to the position indicated by the dotted lines, by means of a worm gear, when it is released and allowed to fall and fracture the test piece.

In its upward swing the pendulum carries the pointer *P* over the semi-circular scale *S*, graduated in degrees. Corresponding to the angle through which the pointer is carried, the energy absorbed in breaking the specimen is read from a table.

The tapped hole at the back of the disc is intended to receive specimens for tension-impact tests; a special attachment is provided for the purpose. This test is not standardized and very little appears to have been published concerning it.

TABLE X

Dimensions of Impact Machines

Machine	Type	Height of Fall		Distance between Supports		Velocity of Impact		Striking Energy	
		m.	ft.	mm.	in.	m. per sec.	ft. per sec.	kg.-m.	ft.-lb.
Frémont	Falling tup.	4·0	13·1	22	0·865	8·86	29	—	—
Charpy (large)	Pendulum	3·144	10·3	120	4·73	9·68	32	300	2 170
Charpy (small)	,,	1·33	4·36	40	1·58	5·53	17·33	30	217
Izod (large).	,,	0·61	2·0	22	0·865	3·5	11·4	16·6	120
Izod (small)	,,	—	—	—	—	3·05	10	3·2	23
Guillery (rotating disc) . .	,,	—	—	40	1·58	8·86	29	60	434

The striking velocity of the small Charpy machine is 5·28 metres per sec., i.e. 17 ft. 4 in. per sec.

The Izod Machine. The Izod machine is similar in principle, but the specimen is tested as a cantilever. The knife of the hammer is horizontal and strikes the specimen at a point 22 mm. above the plane of fixing (Fig. 119). The test piece is gripped by a vice and a gauge is provided to aid in setting it correctly in position. The energy absorbed in fracture is read off directly from a scale. The striking velocity is 11·4 ft. per sec.

The 30 kg.-m. Charpy and the Izod 120 ft.-lb. machines both take a specimen 10 × 10 mm. in section. The Charpy machine is used on the Continent. Some experimenters prefer this machine on account of the test piece not being stressed in the region of the notch by the grip of a vice. However, the Izod machine is the one adopted in this country for standard tests and both machines give comparable results below about 60 ft.-lb.

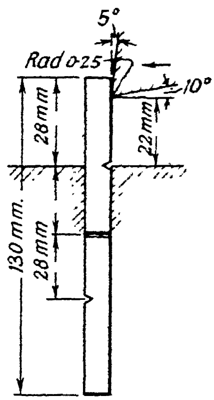


FIG. 119. METHOD OF GRIPPING AND STRIKING TEST PIECE IN IZOD MACHINE

The Amsler Machine. A machine designed for making both Izod and Charpy tests is made by Messrs. Amsler of Schaffhouse. The upward swing of the hammer causes a cursor to travel over a vertical scale which indicates directly the energy absorbed in fracturing the test piece.

A rope having one end attached to the hammer and the other end wrapped round a spindle on which it can slip under the action of a weight prevents the hammer from falling back after its upward swing. The brake is so arranged that it has no effect on the upward swing of the pendulum.

In making a test the hammer is raised until it engages with a hook on the suspension frame. The hook is disengaged by hand when it is desired to release the hammer.

The striking edge of the hammer is fixed at the centre of percussion so that there is practically no vibration of the pendulum when the bar is struck and no energy absorbed by it.

The striking edge has an included angle of 45° and is rounded to a radius of 3 mm.

Then

$$\text{Initial energy} = \mathrm{WR}(1 - \cos \alpha)$$
$$\text{Final energy} = \mathrm{WR}(1 - \cos \beta)$$
$$\text{Friction loss on downward swing} = (\mathrm{WR}/2)(\cos \phi - \cos \alpha)$$
$$\text{Friction loss on upward swing} = (\mathrm{WR}/2)(\cos \beta - \cos \theta)$$

The energy in pendulum just before impact

$$= \mathrm{WR}(1 - \cos \alpha) - (\mathrm{WR}/2)(\cos \phi - \cos \alpha)$$
$$= (\mathrm{WR}/2)(2 - \cos \alpha - \cos \phi)$$

The energy in pendulum just after fracture

$$= \mathrm{WR}(1 - \cos \beta) + (\mathrm{WR}/2)(\cos \beta - \cos \theta)$$
$$= (\mathrm{WR}/2)(2 - \cos \beta - \cos \theta)$$

If ω is the angular velocity after fracture the kinetic energy of the pendulum $= \mathrm{WK}^2\omega^2/2g = \mathrm{WV}^2/2g$

and $$(\mathrm{V}^2/2g) = (\mathrm{R}/2)\ (2 - \cos \beta - \cos \theta)$$

Assuming the test piece to move with the velocity V, its kinetic energy is

$$w\mathrm{V}^2/2g = (w\mathrm{R}/2)(2 - \cos \beta - \cos \theta)$$

Hence, energy absorbed in fracture

$$= (\mathrm{WR}/2)(2 - \cos \alpha - \cos \phi) - (\mathrm{WR}/2)(2 - \cos \beta - \cos \theta)$$
$$-(w\mathrm{R}/2)(2 - \cos \beta - \cos \theta)$$
$$= (\mathrm{WR}/2)(\cos \beta + \cos \theta - \cos \alpha - \cos \phi)$$
$$-(w\mathrm{R}/2)(2 - \cos \beta - \cos \theta)$$

With no friction this becomes

$$\mathrm{WR}(\cos \beta - \cos \alpha) - w\mathrm{R}(1 - \cos \beta)$$

and neglecting the kinetic energy of the test piece,

$$\mathrm{WR}(\cos \beta - \cos \alpha)$$

In practice a table of impact values is used which gives the energy absorbed corresponding to any angle of rise β, or a specially calibrated scale is provided.

If it is desired to calibrate the machine this may be done by releasing the pendulum from an angle of inclination $\phi \overline{\overline{<}} \alpha$ and noting the angle of ascent β_1. The pendulum is allowed to make another forward swing freely and the angle of ascent β_2

again noted. The mean energy absorbed in an upward swing is $(WR/4)(\cos \beta_2 - \cos \beta_1)$ and represents the friction loss for an angle of ascent $(\beta_1 + \beta_2)/2$.

A repetition of the process enables a curve to be plotted to give the friction loss for any angle of rise.

Another method is to measure the angle of inclination θ of the pendulum before release by means of a protractor of the Starrett or similar type and to note the corresponding angle of rise β.

The loss by friction for the upward swing is then

$$(WR/2)(\cos \beta - \cos \theta)$$

As in actual testing the angle α is constant, the friction loss on the downward swing will be constant and can be determined by the method already described. To determine the constants of the pendulum accurately necessitates the dismantling of the machine, but the product WR of the above formula may be checked by supporting the pendulum in a horizontal position by a spring balance attached to the hammer.

The Guillery Machine. The Guillery machine consists of a totally enclosed steel flywheel rotated by hand. The speed of rotation is indicated by the height of a coloured liquid in a vertical glass tube. When the desired speed has been obtained, indicated by the liquid tachometer as corresponding to a rim speed of 29 ft. per sec., a sliding knife in the rim of the wheel is released. The knife strikes and fractures the test piece which is held in a die at the side of the machine. The speed of the wheel falls by an amount depending on the energy absorbed in fracturing the test piece, and this is indicated by a fall in the level of the liquid in the tube. The tube is calibrated so that the amount of energy can be read off directly.

Influence of the Shape of the Notch. Doubt has been thrown on the impact test as being incapable of giving consistent results, but it has been established that variations in results are due, not to the mode of testing, but to the lack of homogeneity in the material tested.

Tests by M. Charpy showed that by taking especial care in the heat treatment to ensure homogeneity of structure, it is possible to obtain a degree of uniformity in the results of notched-bar tests which is higher than in any other mechanical test to which the material can be subjected.

The efficacy of the notch in distinguishing the effect of heat

treatment is shown by an example given by Sir Robert Hadfield.

A steel of composition

C	Si	Mn	S	P	
0·12	0·02	0·28	0·02	0·02	per cent

after quenching and tempering possessed the following mechanical properties—

Elastic limit . .	16 tons per in.^{2}
Tenacity . . .	28 tons per in.^{2}
Elongation . .	35 per cent
Reduction of area .	65 per cent

Under the Frémont test the specimen bent double cold.

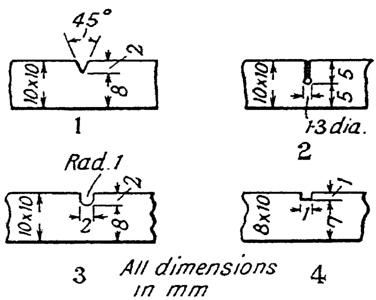


FIG. 121. TYPES OF NOTCH

Portions of the same steel were heated to about 1 200° C. and allowed to cool slowly.

The results then obtained were

Elastic limit . .	9 tons per in.^{2}
Tenacity . . .	22 tons per in.^{2}
Elongation . .	46 per cent
Reduction of area .	64 per cent

Under the impact test this steel was found to be extremely brittle; it snapped almost like cast iron and did not bend 1°.

So far as the notch itself is concerned, any form can be used independently of the type of machine, though not necessarily with advantage.

The standard V-notch 2 mm. deep and 0·25 mm. radius at the root is more economically produced than the Charpy standard having a $1\frac{1}{3}$ mm. drilled hole, and can be easily formed by a suitable milling cutter. Several forms of notch are shown in Fig. 121, namely (1) Izod, (2) Charpy, (3) Mésnager and (4)

Frémont. The standard notch adopted by the British Standards Institution is the Izod 2 mm. deep, 45° included angle with a root radius of 0·25 mm. The standard Izod specimen is shown in Fig. 122. The figure also shows a round form with a straight notch. A curved notch is also used (B.S.S. No. 131). As it is customary to make at least three tests on a sample, the Izod

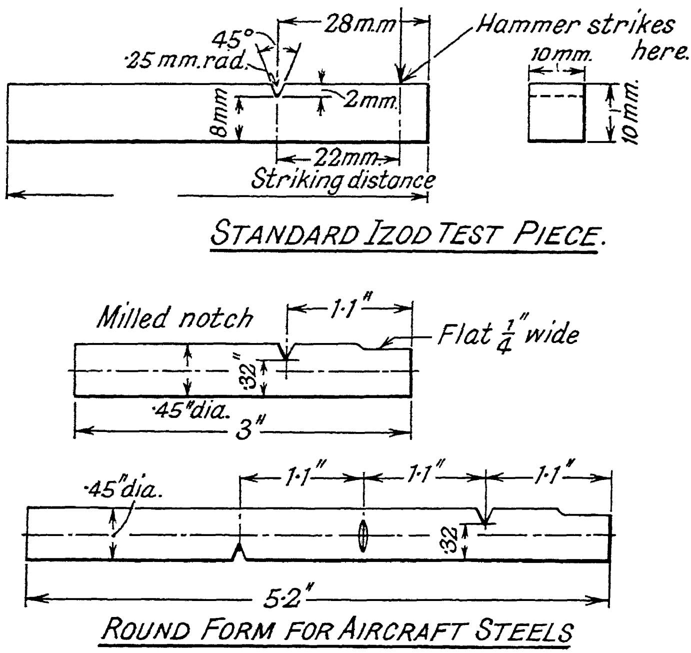


FIG. 122. STANDARD IZOD TEST PIECES

test piece is often provided with three notches arranged so that tests can be made in different directions relative to the cross-section.

Messrs. Greaves and Moore state that considerable rounding of the notch, as in the Mésnager form, seriously reduces its capacity for distinguishing between tough and brittle material.

According to Petrenko, Bureau of Standards Technological Paper No. 289, the Izod and Mésnager notches are about equally efficient, while the Charpy form shows less difference in the notched-bar characteristics of materials. The Charpy notch gives larger values than the Izod for brittle materials

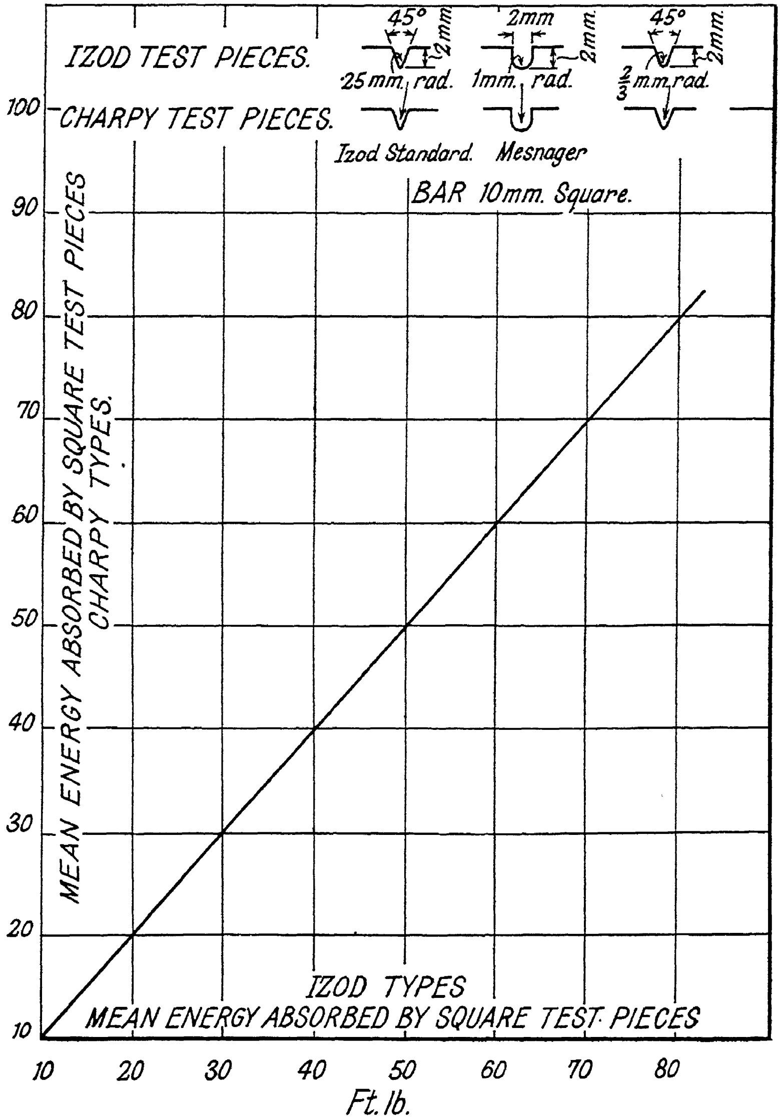


FIG. 123. COMPARISON BETWEEN IZOD AND CHARPY TESTS
(*Philpot*)

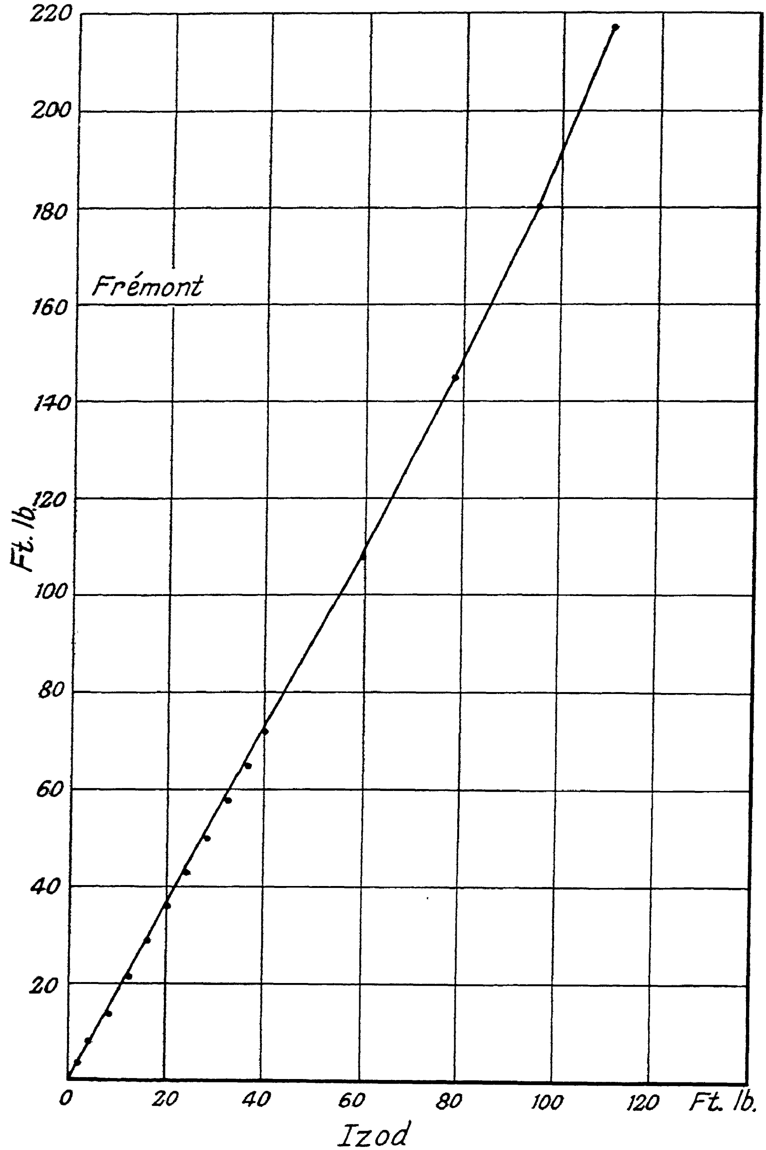


Fig. 124. Conversion Curve
Frémont and Izod tests.
(*Hadfield*)

owing to the larger root radius, but for tough materials it gives smaller values owing to the reduced thickness of specimen. The Izod notch is stated to be preferable for brittle materials.

A point of supreme importance in notched-bar testing is the relation between test results obtained for the same materials on different machines. Unfortunately, in the present state of knowledge, no general relation can be given which will cover all

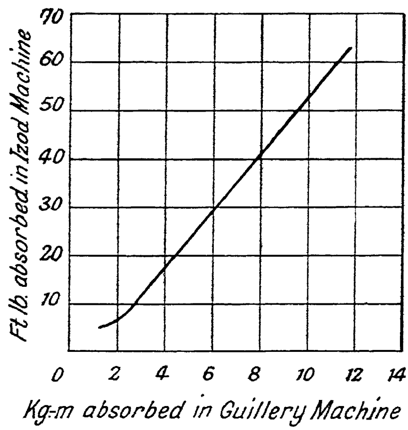


FIG. 125. CONVERSION CURVE
Guillery and Izod tests.
(*Greaves*)

classes of material. However, over a limited range simple relationships hold and particulars of such as are available are given below.

Tests by Philpot, Fig. 123, show that with a standard 10 × 10 mm., 45° notch, 2 mm. deep and 0·25 mm. root radius, Charpy and Izod machines give similar results. Above 70 ft.-lb., the values from the Charpy machine tend to be much higher. For instance, with mild steel giving 140 ft.-lb. Charpy the difference amounts to about 40 ft.-lb.

The curve in Fig. 124 shows the relation between the Frémont and Izod tests. Up to 60 ft.-lb. Izod the relation is

$$\text{Frémont} = 2 \times \text{Izod}$$

The curve in Fig. 125 shows the relation between the Mésnager form tested in the Guillery machine and the standard Izod form in the 120 ft.-lb. Izod machine.

Between 10 and 60 ft.-lb. Izod the relation is

$$\text{Guillery} = 0{\cdot}17 \text{ Izod (ft.-lb.)} + 1$$

for steels of the following types—

Percentage Composition							Yield Point Tons per in.²	Max. Load Tons per in.²
C	Si	Mn	S	P	Ni	Cr		
0·3 to 0·4	0·15	0·65	0·04	0·04	—	—	19 to 24	34 to 44
0·3 to 0·4	0·15	0·6	0·04	0·04	3·5	—	27 to 36	42 to 55
0·3 to 0·4	0·15	0·4	0·04	0·04	3·5	0·6	30 to 40	42 to 52

Tests in an Izod machine using steels of the type—

Percentage Composition							Yield Point Tons per in.²	Max. Load Tons per in.²
C	Si	Mn	S	P	Ni	Cr		
0·35	0·15	0·4	0·03	0·03	3·5	0·8	over 40	55 to 60

give Izod (Mésnager milled notch) = Izod (Standard) + 14·3.

Over 55 ft.-lb. the constant is 8·5.

The above relations are deduced from tests by Greaves and Moore.

For aircraft steels, tests by Philpot show that with the Izod machine.

$$\text{Charpy (Mésnager)} = \text{Standard} + 12 \text{ ft.-lb.}$$

$$\text{Charpy standard } (\tfrac{2}{3} \text{ mm. radius notch}) = \text{Standard} + 7 \text{ ft.-lb.}$$

Impact values for some representative steels are given in Table XXII.

Amsler Universal Hammer Machine. For making repeated impact tests Messrs. Amsler supply a universal hammer machine which can be used for making transverse bending, compression or tension tests. (Fig. 126.) The impact action is produced by hammer blows of uniform strength, made in

rapid succession against the test piece until fracture or a predetermined deformation is reached.

The machine consists of a frame in which is mounted a

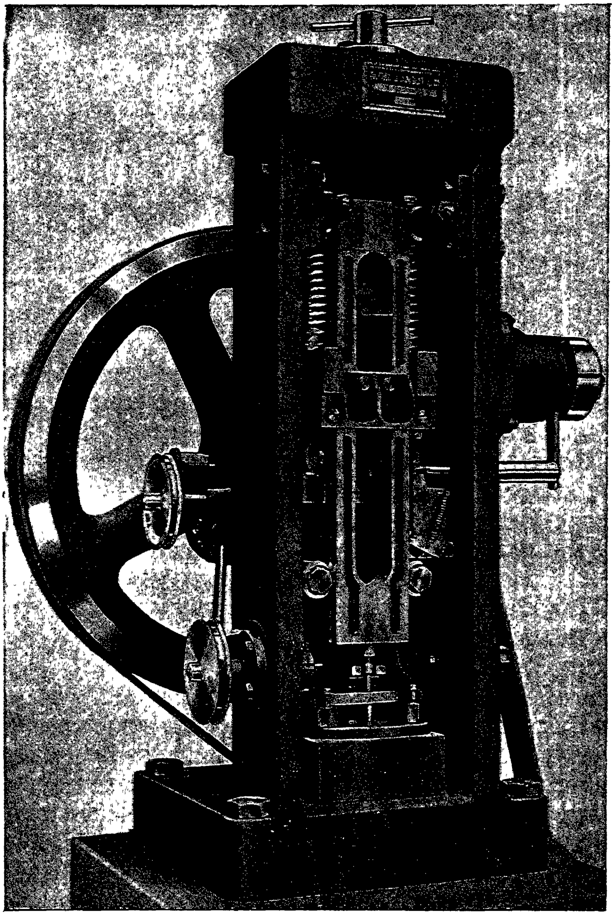


FIG. 126. AMSLER UNIVERSAL HAMMER MACHINE
(Alfred J. Amsler & Co.)

vertically reciprocating hammer having at its lower end a cylindrical steel block for striking the test piece in the compression and bending tests, and at the upper end the arrangement for striking the tensile specimen. The reciprocating motion of the

while the one that is moving upwards will be about to engage with it. Due, however, to the elasticity of the material under test the hammer rebounds after the blow with an upward velocity nearly the same as that at the instant of striking, and as a result the upward moving crank-pin meets the crosshead at a very moderate relative speed. Should it fail to do so, for any reason whatever, the elastic coupling in the flywheel is brought into action and prevents damage.

When a test bar breaks, the hammer continues its downward movement until it disengages the trip mechanism.

The kinetic energy of the blow can be calculated from the expression $wv^2/2g$.

For bending impact tests a round bar from 12 mm. ($\frac{15}{32}$ in.) to 16 mm. ($\frac{5}{8}$ in.) and 12 cm., say $4\frac{3}{4}$ in. long is used. The test bar is placed on two rests, the edges of which are 10 cm. ($3\frac{15}{16}$ in.) apart, and it is held down against the rests by two light springs. End stops are provided to prevent lateral movement and the rests can be adjusted to suit the thickness of the test bar.

Whilst the test bar is being struck, it either rotates about its axis at constant speed so that the blows are uniformly distributed over its circumference, or it is turned through 180° after each blow. It is then struck on opposite sides alternatively and bent to and fro until it breaks.

For compression impact tests the specimen, which should have plane end surfaces and a length of about $2\frac{3}{4}$ in., stands upright on a block and rests against a vertical support to which it is lightly held by a spring.

For tensile impact tests round bars with threaded ends are generally used. One end of the bar is passed through an eye at the top of the machine frame and the other end through the upper part of the hammer. Hardened steel nuts are screwed on to both ends of the test bar in such a manner that the bar is suspended by its upper end from the machine frame while the nut at the lower end is in a position to be struck by the hammer in its descent.

To prevent breakage inside the grips it is necessary to reduce the section of the bar between the threaded ends. Tensile test pieces are 11 cm. ($4\frac{5}{16}$ in.) long and the ends provided with a $\frac{3}{4}$ in. gas thread. The free length between the grips is about 3 in.

The machine is rather noisy in action and this fact should be considered when choosing a site for its installation.

Repealed Impact Tests. The results of some tests with the Amsler Repeated Impact Machine are given below, by kind permission of Messrs. High Duty Alloys, Ltd.

1st Series. Energy of blow . . . . . . 1·16 ft.-lb.
Number of blows per minute . . 620

Test	R.R.56	"Y" Alloy	Duralumin
1	4 770	170	160
2	6 320	210	150
3	6 335	200	150
4	5 000	210	145
5	4 510	165	150
6	6 820	165	150
Average	5 626	187	151

2nd Series. Energy of blow . . . . . . 0·517 ft.-lb.
Number of blows per minute . . 620

Test	R.R.56	"Y" Alloy	Duralumin
1	103 770	53 030	34 440
2	98 000	70 050	39 820
3	100 500	65 850	39 420
4	107 000	52 500	32 450
5	105 360	58 500	41 760
6	90 150	—	—
Average	100 832	50 038	37 587

3rd Series. Energy of blow . . . . . . 0·351 ft.-lb.
Number of blows per minute . . 340

Test	R.R.56	"Y" Alloy	Duralumin
1	326 100	113 180	99 450
2	230 390	69 980	67 580
3	370 920	159 920	90 440
4	207 100	148 520	115 100
5	197 970	107 870	72 000
6	324 820	105 780	64 870
Average	276 218	117 541	84 906

4TH SERIES. Energy of blow . . . . . . 0·156 ft.-lb.
Number of blows per minute . . 340

Test	R.R.56	"Y" Alloy	Duralumin
1	1 000 000	255 460	226 430
2	1 000 000	255 940	333 970
3	1 000 000	478 800	195 620
4	1 000 000	148 470	247 530
5	1 923 000	619 150	224 280
6	All above unbroken	361 840	248 440
Average	—	353 276	246 045

CHAPTER X

REPEATED STRESSES

Fatigue of Metals. The extensive series of tests carried out by Wöhler between the years 1860–1870 on the effect of repeated stresses on materials showed clearly that a completely reversed stress much lower than the ultimate strength of the material, or lower even than the yield point as determined by the tensile test, could cause fracture of a steel specimen if only the application of stress was repeated a sufficient number of times. On the other hand, below a certain maximum value of the stress Wöhler showed that the number of repetitions of stress might be indefinitely large without causing rupture.

The phenomenon of fracture under repeated stressing is termed *fatigue.*

Formerly, it was held that under repeated stress a metal developed a crystalline structure, but later metallographic research has shown that metals are themselves built up of crystalline grains and that, therefore, crystallization of the material is not a consequence of repeated stressing. (See Chapter II.)

The impression that crystallization was produced in a metal by the action of repeated stresses arose largely from the brittle appearance of such fractures, and from the fact that many of these exhibited a coarse crystalline structure. This appearance may have been due to the heat treatment received by the material during the process of manufacture.

A mild steel specimen when tested in tension suffers considerable plastic flow prior to fracture and the surface of the ruptured section shows a silky, fibrous structure, owing to the crystals having stretched in the direction of the pull. A fatigue crack, however, has an entirely different appearance. Owing to a local defect, or to the action of fatigue in causing hair cracks to form in the material, the concentration of stress at the end of the crack under the stressing action causes the crack to spread progressively until the cross section becomes so reduced that the remaining portion fractures suddenly under the load imposed.

In general, fatigue failures of ductile materials show two distinct zones; one exhibiting a characteristic brittle appearance

—the fatigue fracture proper; the other exhibiting a more or less ductile fracture somewhat similar to that shown under a tension test. The difference between the two zones is not, as a rule, so apparent in the more brittle materials. Fatigue fractures are sometimes discoloured by chemical action. Fig. 127 shows a fatigue fracture of a mild steel test piece.

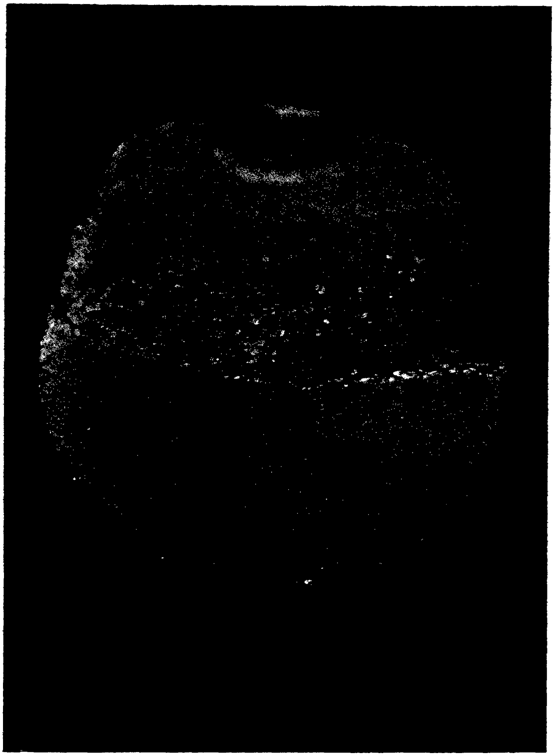


FIG. 127. FATIGUE FRACTURE OF MILD STEEL TEST PIECE
(Bruntons (Musselburgh) Ltd.)

The shape of the zones appears to depend on circumstances attendant on the material and the mode of stressing. Professor Bacon has classified the fatigue fractures met in practice as follows (see Fig. 128)—

(*a*) Concentric.
(*b*) Eccentric.
(*c*) Double sided, convex.
(*d*) Double sided, concave.
(*e*) Single sided, concave.
(*f*) Single sided, convex.

In the diagrams the white portions represent the fatigue zones and the black portions the ruptured core.

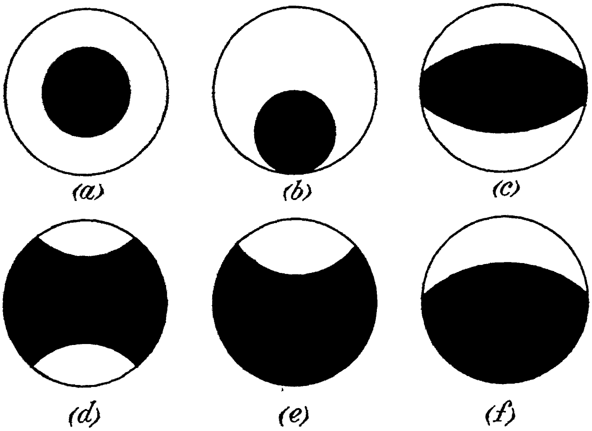


FIG. 128. ILLUSTRATING TYPES OF FATIGUE FRACTURE

In some cases many fatigue zones are apparent. The arcs separating the respective zones are generally elliptical in shape. The diagram in Fig. 129 represents a Hypothetical Crack Progress Chart, which consists of a system of elliptic arcs starting from a small semicircle centred at the origin of the crack at A and eventually straightening out into a diameter tilted back through 15° to the direction of rotation. Later work indicates that high stresses produce concentric fractures while low stresses produce eccentric fractures.

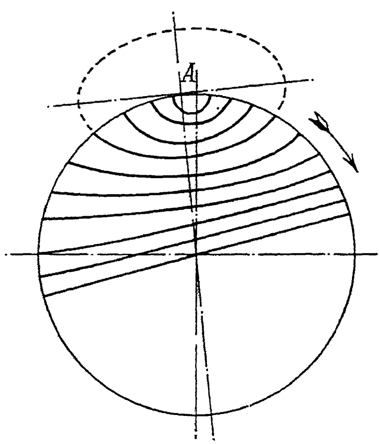


FIG. 129. HYPOTHETICAL CHART SHOWING PROGRESS OF A FATIGUE CRACK

In rotating bar test pieces the maximum stress occurs at the outer fibres, and the fatigue crack usually starts at the periphery and spreads towards the centre. When there is concentration of stress due to fillets or holes, the crack

usually starts at the most highly stressed portion and spreads from this point.

Work of Bauschinger and Bairstow. One of the first investigators of cycles of stress was Bauschinger, a contemporary of Wöhler. He loaded and unloaded specimens slowly and determined the stress-strain relation under these conditions by using a very sensitive extensometer. He showed from his tests that the proportional limits in tension and compression are not fixed points for a given material and that they may be displaced by submitting a specimen to cycles of stress.

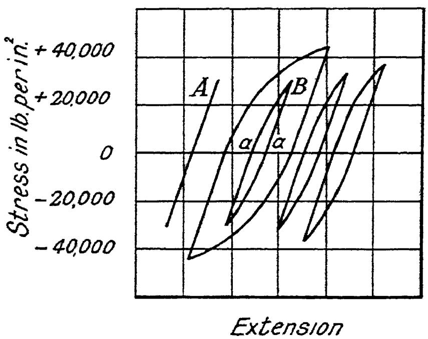


Fig. 130. Hysteresis Loops Obtained Under Cyclical Stress

To explain why the endurance limit for steel under reversed stress was lower than the limit of proportionality obtained in the static test, Bauschinger advanced the theory that the material as received from the manufacturer might have its proportional limits in tension and compression raised by cold work, and that the true or natural proportional limits are those which are established after submitting the material to cycles of stress. These natural proportional limits are supposed to define the safe range of stress in fatigue tests.

The work of Bauschinger was extended by Bairstow, who used a slow loading and unloading machine (2 cycles per minute) with a mirror extensometer, and obtained the stress-strain relations for cycles with various ranges of stress. Some of Bairstow's results are shown in Fig. 130. The material was axle steel possessing a yield point of 50 000 lb. per in.^{2} and an ultimate strength of 84 000 lb. per in.^{2} The cycle consisted of equal reversed stresses. The line A represents the initial tension—compression test with the range 31 400 lb. per in.^{2}, within these limits the straight line relationship holds.

The specimen was next subjected to cycles of reversed stress of 31 400 lb. per in.^{2} when it was found that the straight line developed into a loop. Curve B shows the loop obtained after 18 750 cycles. It will be noted that in this case the initial

proportional limits are higher than the so-called "natural" limits obtained after many cycles of reversed stress, and that a cyclical permanent set of width *aa* was produced. The remaining loops were obtained after a number of cycles of reversed stress equal to 33 500, 37 500 and 47 000 lb. per in.^{2} respectively, sufficient apparently to stabilize the size of the loops.

When the width of these loops was plotted against the corresponding maximum stress Bairstow found that the results of these experiments gave approximately a straight line. The intersection of this line with the stress axis determines the range of stress at which there is no looping effect.

The range of stress so defined was assumed to be the safe range of stress and subsequent endurance tests have tended to verify this assumption.

Mechanism of Fracture. The first attempt to explain the mechanism of fracture in endurance tests was made by Ewing and Humfrey. They used a rotating specimen of Swedish iron with a polished surface and examined this surface microscopically after applying cycles of reversed stress. They found that if stresses above a certain limit were applied, slip bands appeared on the surface of some of the crystals after a number of cycles. The number of slip bands increased as the cycles were repeated and some of the previous slip bands seemed to broaden out. This broadening process continued until a crack occurred, the crack following the marking of the broadened slip bands. They found that a reversed stress of 11 800 lb. per in.^{2} could be applied millions of times without producing any slip bands. On the basis of their investigations they advanced the theory that cycles of stress which are above the safe range produce slip bands in the individual crystals. If the application of stress cycles is contined, sliding along the surfaces takes place accompanied by friction. According to the theory, as a result of this friction the material gradually wears along the surface of sliding and a crack results. Further investigation showed that slip bands occurred at stresses much lower than the endurance limit of the material. They may develop and broaden without leading to the formation of a crack. The appearance of slip bands cannot therefore be taken as a criterion for determining the endurance limit.

A vast amount of data concerning strength in fatigue has been accumulated in recent years, but up to the present no theory has been established which is wholly satisfactory to

engineers in its explanation of the cause and mechanism of fatigue failure.

Stress Cycles. Fatigue Limit. The type of stressing producing fatigue fractures may be tension and compression, bending, shear, torsion or a combination of these. The cycle of stress need not consist of reversed tension and compression stresses of equal magnitudes. For example, the cycle may vary from a stress of 5 tons per in.^{2} in tension to 10 tons per in.^{2} in compression, or the stress may be wholly tensile but of variable magnitude; or, again, the stress may vary in torsion from a given value to a different value in the opposite direction.

Frequently, both in practice and under test conditions, the magnitude of the stress when plotted on a time base gives a sine curve. Even if this is not the case the stress variation occurs in cycles and it is customary to refer to the repeated variation as a *stress-cycle*.

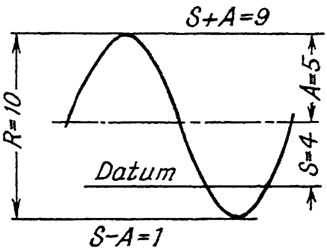


FIG. 131. SIMPLE HARMONIC STRESS CYCLE

The stress throughout the cycle may be regarded as made up of a mean steady stress S and an alternating stress A. The diagram, Fig. 131, illustrates a simple harmonic stress cycle composed of a steady stress $S = 4$ tons per in.^{2} combined with an alternating stress $A = 5$ tons per in.^{2}, giving 9 tons per in.^{2} in tension and 1 ton per in.^{2} in compression. If R be the algebraical difference between the maximum and minimum values of the stress then R is termed the *range of stress*. The limiting values of the stress are thus $S + A$ and $S - A$.

The range of stress within which an indefinitely large number of repetitions of stress will not cause fracture is called the *fatigue range*. The *fatigue limit* is the greatest stress that can be applied in a given stress cycle without eventually causing fracture.

The *endurance* is the number of cycles required to cause fracture when the range of stress is maintained invariable throughout the test. The fatigue limit is by some authorities referred to as the *endurance limit*.

The value of the fatigue limit appears to be independent of the speed or frequency, at least up to a few thousands of cycles per minute. Excessively high frequencies may tend to cause

For example, if L = 2 in., $d = \frac{5}{16}$ in.; $r = \frac{5}{8}$ in., then $x =$ 0·0163 in. less than 1 per cent of the nominal length. If L = 4 in.; $d = \frac{1}{2}$ in.; $r = \frac{3}{8}$ in.: then $x =$ 0·0078 in., which is less than 0·2 per cent of the nominal length.

The smaller the radius of the fillet the more nearly will the section at which the stress is a maximum coincide with the section of the junction, but this is attended with some disadvantage owing to stress concentration causing the value of the fatigue limit to be different from that calculated from the ordinary formula for bending. For comparative purposes it may be of less moment; in fact, one investigator at least has used a fillet radius of one-tenth diameter of the test piece.

The effect of the radius of the fillet has been investigated experimentally, and some results for carbon steel are given in the following tables—

TABLE XI

EFFECT OF RADIUS OF FILLET ON FATIGUE PROPERTIES OF 0·49 PER CENT CARBON STEEL SPECIMENS

(Moore)

Radius of Fillet r in.	Minimum Diameter of Specimen d in.	Ratio r/d	Fatigue Limit lb. per in.²	Reduction in Fatigue Limit %
9·85	0·275	36	49 000	—
1·00		3·5	47 500	3
0·275		1	44 500	9
0·00		0	24 000	51

TABLE XII

EFFECT OF RADIUS OF FILLET ON FATIGUE PROPERTIES OF 0·33 PER CENT CARBON STEEL SPECIMENS

(Timoshenko and Lessels)

Radius of Fillet r in.	Minimum Diameter of Specimen d in.	Ratio r/d	Fatigue Limit lb. per in.²	Reduction in Fatigue Limit %
Standard Form	—	—	32 000	—
0·15	0·6 parallel	1/4	29 900	6·5
0·05	0·6 parallel	1/12	21 000	34

These results show the influence of change of section on the value of the fatigue limit as experimentally determined, particularly when the ratio r/d falls below unity. The point is important in relation to any proposed standardization of the Wöhler test piece.

A method of securing a more uniform distribution of stress along the test piece is to employ the tapered form of Fig. 133.

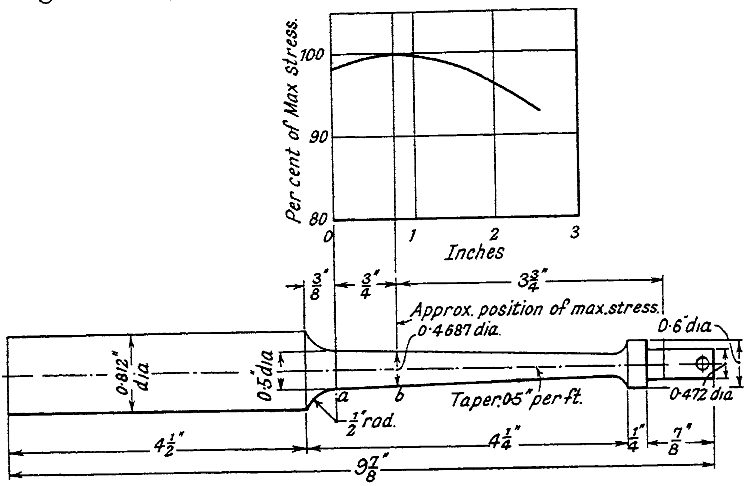


FIG. 133. WÖHLER TEST PIECE
Tapered form.

The taper is 0·5 in. per ft. and the variation of fibre stress over the length ab is only about 1 per cent. The variation of stress is shown in the graph in the same figure.

Specimens are sometimes made hollow at the shank end. The method is expensive as the bore needs to be finished smooth.

Another objection urged against the Wöhler test piece with single point loading (that is, with the test piece loaded as a cantilever) is that in addition to the bending stress there is also a direct shear on the section. This objection is overcome by the use of four-point loading in which the specimen is supported at the ends and loaded at two points equidistant from the supports. Fig. 134. This method of loading gives a constant bending moment over the gauge length with no direct shear on the section. A larger amount of material is required with this form of specimen unless the holders are designed to accommodate a short test piece. A modification of the foregoing is to

use the Wöhler test piece with two-point loading, Fig. 135. The downward load is applied by a dead-weight and an equal and opposite load is applied by means of a small single-lever testing machine. The bending moment $w \times l$ is constant over

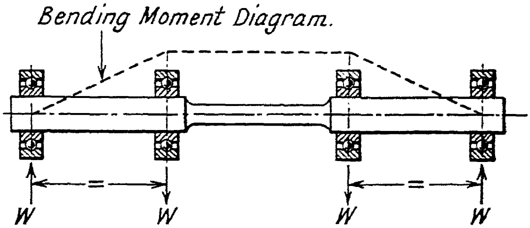


FIG. 134. SONDERICKER TEST PIECE
Four-point loading.

the test portion. In any case the amount of direct shear is very small. A $\frac{5}{16}$ in. diameter specimen 2 in. long carrying a single load of 40 lb. at its free end is stressed in bending to about 12 tons per in.^{2}, whilst the mean shear stress over the section is only 500 lb. per in.^{2}, which is less than 2 per cent of the bending stress.

The degree of finish given to specimens is also of importance. Kommers found that test pieces of ordinary mild steel finished

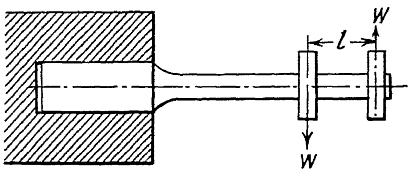


FIG. 135. WÖHLER TEST PIECE
Two-point loading.

by hand filing, had a 33 per cent longer life than when merely turned, and that this increase amounts to about 48 per cent when specimens are polished.

The oft-quoted results of Professor H. Norman Thomas are further confirmation of this. Gelatine casts were taken from a number of specimens of various degrees of surface finish. The casts were sliced with a microtome and the depth of scratch and radius of curvature at the root measured by the aid of a projection apparatus. The results of fatigue tests on scratched and highly polished specimens were compared, and comparison was also made with deductions from mathematical theory.

TABLE XIII

EFFECT OF SURFACE FINISH ON THE ENDURANCE OF MILD STEEL

Type of Finish	Estimated Reduction in Endurance Limit %
Turned	12
Coarse file	18–20
Bastard file	14
Smooth file	7½
No. 3 emery	6
No. 1 emery	4
No. 0 emery	2–3
Fine carborundum	2–3
Fine ground	4
Accidental scratches (maximum found)	16

Endurance Curves. A typical endurance curve for carbon steel is given in Fig. 136. The curve tends to show that at a sufficiently low stress the material would withstand an in-

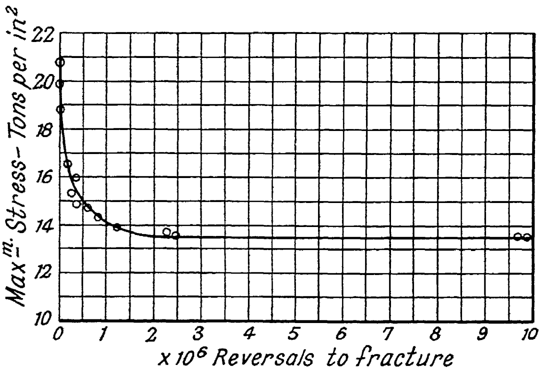


FIG. 136. ENDURANCE TEST ON 0·83 PER CENT CARBON STEEL
(*Moore*)

definite number of reversals. For some steels, however, and for non-ferrous materials the endurance curves approach the axis of abscissae very slowly, indicating that any varying load, however small, would ultimately cause fracture of the material. Fatigue tests in which there is a combination of the effects of stress and corrosion lead to a similar result. It should be remarked, in passing, that failure of a metal under repeated

stress is rapidly accelerated when, in addition, corrosive influences are at work. (See Fig. 170.)

Alloy steels, especially those containing a high nickel content, often give no "curve," but show an abrupt change in the region of the fatigue limit.

Fig. 136 refers to some results obtained by Moore on a 0·83 per cent carbon steel, some specimens of which remained unbroken after 100 million reversals of stress. It will be observed that in the neighbourhood of the fatigue limit a small change in

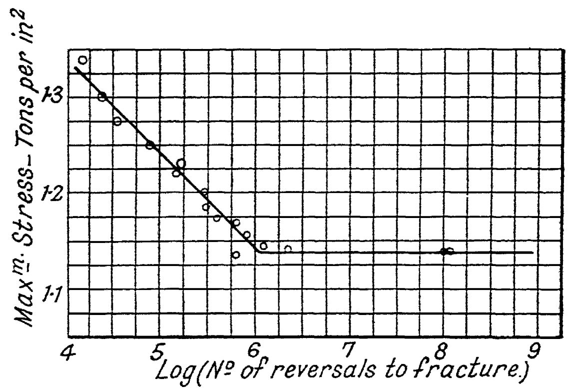


FIG. 137. LOGARITHMIC PLOT OF TEST OF FIG. 136

the stress has considerable influence on the life of the material. The fatigue limit is about 13·4 tons per in.^{2}; with an increase of 3 per cent in the stress the specimens would have failed, probably at one million reversals.

Another method of plotting test results is to use logarithmic or semi-logarithmic co-ordinates.

Fig. 137 is plotted from the results already given in Fig. 136; the logarithms of the stresses being plotted against the logarithms of the numbers of reversals. The abrupt change at the fatigue limit is particularly noticeable.

The ratio of the fatigue limit to the ultimate strength in the case of steels varies from about 0·35 to about 0·65.

Wöhler Fatigue Testing Machine. The general arrangement of a Wöhler machine employing single-point loading is shown in Fig. 138. Two specimens can be tested at once and the machine can be arranged to accommodate specimens of 2 in. and 4 in. or other convenient test lengths. The drive is by electric motor at 2 000 r.p.m. A Veeder counter is provided for each spindle

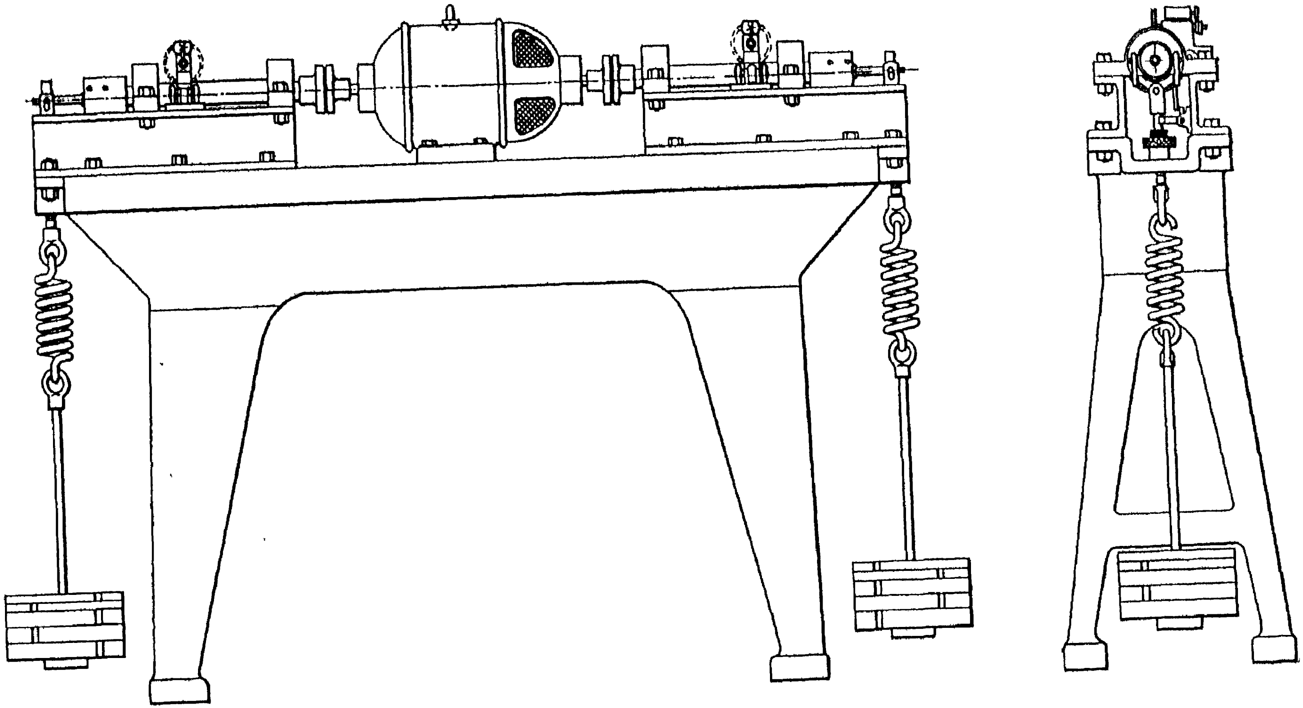


FIG. 138. WÖHLER TESTING MACHINE
(*Machinery*)

and is driven through a 100 to 1 reduction gear. The load is applied by slacking back the hand nut seen in the end view.

A spring or a strip of rubber may be inserted between the deadweight and the ballrace at the end of the specimen to assist in damping out any vibration caused by running, though this is negligible if the specimen is set up accurately. For this purpose an Ames dial may be used. Dashpots are sometimes

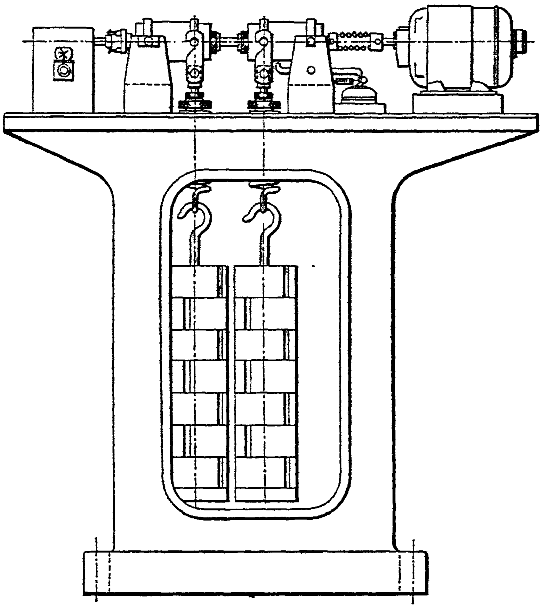


FIG. 139. MACHINE EMPLOYING FOUR-POINT LOADING

employed in place of the damping springs. The counter is tripped when the test piece fractures. Some experimenters view a double ended machine with disfavour and prefer to work with a single spindle.

Author's Fatigue Testing Machine. A machine employing four-point loading, designed by the Author, is shown in Fig. 139. The test specimen is carried between two spindles running in ball bearings mounted in housings pivoted on vertical supports, the drive being through a spring transmission from a high speed motor. The tail spindle communicates its motion to a substantial revolution counter. Both the motor and counter are cut out of action immediately fracture occurs.

A novel feature of the machine is the method of gripping the test piece. This is accomplished by means of special chucks which obviate the necessity for drilling and tapping the ends of the specimens; a distinct advantage when a large number of specimens have to be tested, particularly if they should be of hard or tough material.

Damping springs are provided between the loads and the supporting knife-edges. Provision is made for gradually applying and releasing the loads.

Specimens are 3 in. long and may vary from 0·2 to 0·3 in. diameter at their smallest section. Close limits in machining specimens to length are unnecessary as sufficient freedom of the housings is allowed to accommodate variations in the overall length of the specimens.

The type of specimen adopted and the method of gripping it enables alignment to be obtained with no trouble on the part of the operator. Specimens may be parallel in the mid-portion of their length or turned so that the longitudinal profile is a curve of some 3 in. radius as desired.

A load of 60 lb. with a specimen 0·3 in. diameter will yield a stress of 30·3 tons per in.^{2}, and with a specimen of 0·25 in. diameter will yield a stress of 51·3 tons per in.^{2}

The normal speed of operation is 3 000 r.p.m.

The machine is now made by Messrs. Edward G. Herbert, Ltd.

The Haigh Alternating Stress Testing Machine. The alternating stress testing machine designed by Professor Haigh is made by Messrs. Bruntons Ltd., Musselburgh. Two sizes are made, one having a load range of 1½ tons and the other a load range of 6 tons. The important advantage of this machine lies in the uniform distribution of stress across the full section and along a considerable length of the test piece, and in the possibility of applying stress cycles of unequal plus and minus limits.

The essential features of the machine are shown in Fig. 140. A pair of electromagnets M_1 and M_2 are supplied with two-phase currents from a small alternator of special design. The forces generated by these magnets, pulling alternately on the faces of an armature A, are transmitted to the lower end of the test piece T, the upper end of which is gripped in a holder that forms part of the adjustable head H. The magnets and the adjustable head are rigidly connected by four vertical columns that rise from the base of the machine. The vibrating armature

is guided by springs without the use of lubricated slides. The frequency of reversal of stress is governed by the speed of the alternator, usually 1 000 r.p.m., and the frequency of stress reversals is double this, or 2 000 cycles per min.

Each electrical cycle produces two mechanical cycles, i.e. pull—push—pull—push.

An important feature of the machine is the set of compensating springs *S* that connect the vibrating crosshead below the magnets to the base of the machine.

Although the majority of fatigue tests are performed with loads ranging between equal intensities of push and pull the ratio between the extremes may be varied at will. This adjustment is effected by giving the flat springs *S* a suitable degree of initial load. The same springs serve the important purpose of compensating the force required to accelerate the armature and other vibrating parts.

By means of sliding clamps the spring stiffness may be adjusted to suit widely different frequencies of operation.

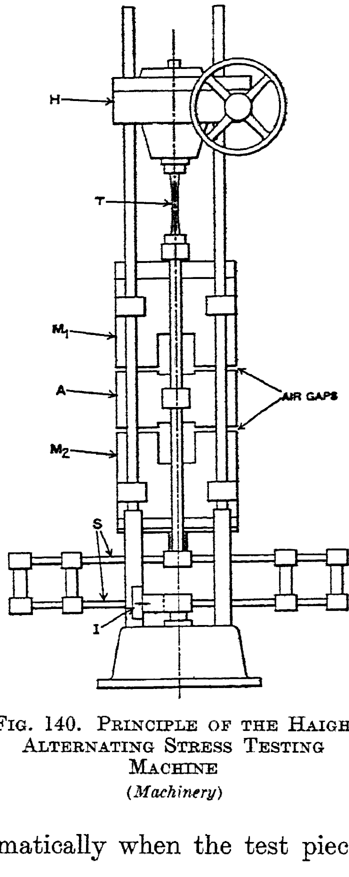


FIG. 140. PRINCIPLE OF THE HAIGH ALTERNATING STRESS TESTING MACHINE
(*Machinery*)

The machine is stopped automatically when the test piece breaks.

The stress meter, Fig. 141, is a soft iron alternating current instrument comprising two independent movements in a single case. The outer, long, uniformly divided scale is used for reading the range of stress applied to the machine. The inner short scale, graduated to read amperes, is arranged with a central zero for the purpose of balancing the currents in the two phases to ensure that the air gaps are equal on the upper and lower faces of the vibrating armature.

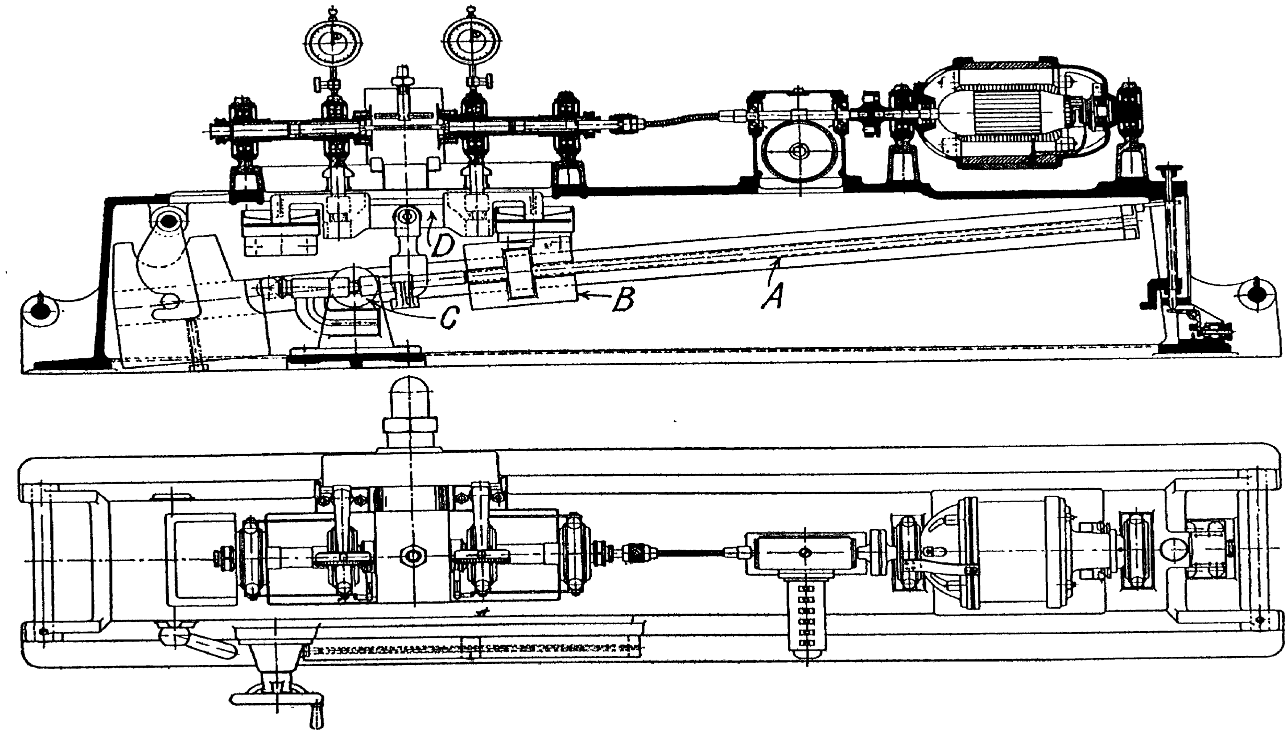


FIG. 153. SCHENK PENDULUM FATIGUE TESTING MACHINE
(*Carl Schenk, Darmstadt*)

very suddenly and turns through an angle approaching 90.° In Fig. 154 curve 1 shows the total energy absorbed per unit volume per cycle as the stress is increased, while curve 2 represents the energy absorbed by the test piece.

The point A at which the curve begins to bend is referred to as the *first characteristic point.* The energy curve rises steeply after this point is passed and a tangent drawn to the curve to intersect the axis of abscissae yields the point B, the *second*

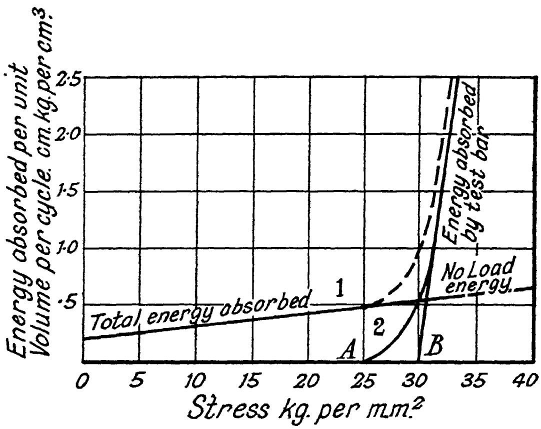


FIG. 154. ILLUSTRATING HYSTERESIS-LOSS METHOD OF TESTING

characteristic point, which for ferrous materials is assumed to indicate the fatigue limit.

The curve of temperature rise is similar to the energy absorption curve.

With non-ferrous materials the fatigue limit falls near to the point A.

Characteristic curves for steel and brass are given in Fig. 155 (a) and (b).

To use the method for short-time fatigue tests comparison should first be made with results obtained by the usual duration tests on material similar to that which it is proposed to test.

Fatigue Tests by Torsional Oscillations. Another machine by Schenk, of Darmstadt, for making fatigue tests employs the method of torsional oscillations. It is shown diagrammatically in Fig. 156. The relation between the applied torque and the angle of twist is indicated on a ground glass screen by an optical method. The test bar forms part of an oscillating system

consisting of a flywheel counterweight at one end and a smaller mass at the other.

The torsional oscillations are effected through an electro-

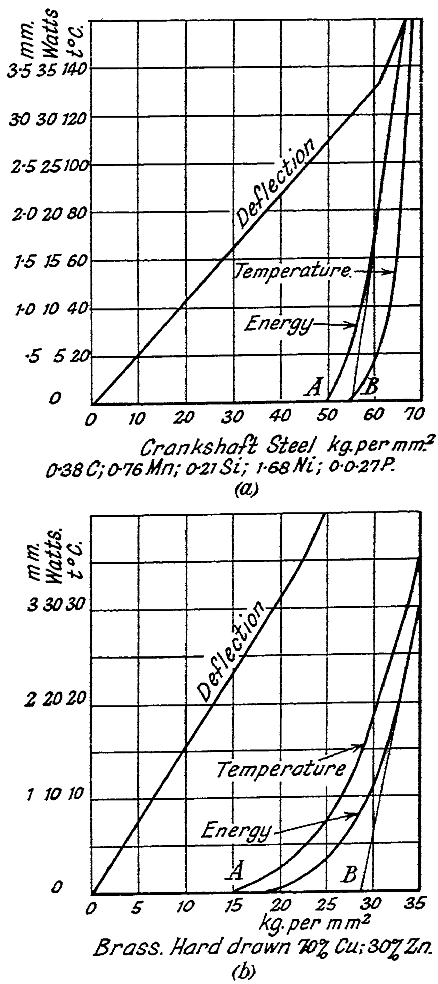


FIG. 155. CURVES OF ENERGY LOSS AND TEMPERATURE RISE

magnetic exciting system. A beam of light from the lamp is projected successively on to two groups of mirrors and thence to the glass screen, where it appears as a point of light. The

position of the spot of light relative to the axes indicates the values of both torque and twist. During a cycle the point of light traverses a closed curve which, owing to the high frequency of the oscillations, appears as a stationary hysteresis loop. The loop which reduces to a straight line when little or no power is absorbed, becomes increasingly inflated as more and more power is absorbed by the test piece. The area of the loop is a measure of the energy absorbed per cycle.

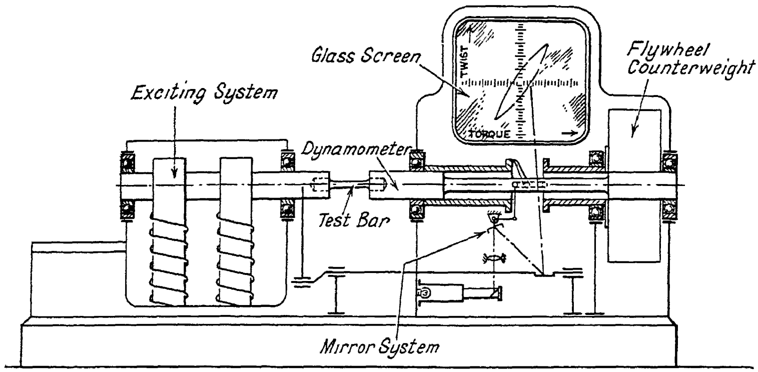


FIG. 156. SCHENK FATIGUE TESTING MACHINE EMPLOYING THE METHOD OF TORSIONAL OSCILLATIONS

Short-time Fatigue Tests. The chief objection to the adoption of the fatigue test in commercial testing is the length of time required to obtain an accurate determination of the fatigue limit. In view of this the search for a short time fatigue test has been prosecuted vigorously. Among the various tests that have been proposed are—

(*a*) Tests based on inelastic action or on mechanical hysteresis.

(*b*) Tests based on rise of temperature.

(*c*) Tests based on deflection of the specimen.

(*d*) Tests based on the energy absorbed per cycle.

(*e*) Tests based on change of electrical resistance as fatigue cracks develop.

The test described on p. 196 is intended to provide a short time method.

G. N. Krouse using an air turbine as a driving motor has constructed a rotating bar machine to run at 30 000 r.p.m. Comparison of tests on various steels, cast iron, brass and duralumin

with tests of the same materials on a machine of the ordinary type running at 1 500 r.p.m. shows that there is no appreciable speed effect up to 10 000 r.p.m. At 30 000 r.p.m. the fatigue limit is raised, in some instances, substantially.

Recently, a test based on repetition of stress combined with tensile tests on the same specimens has been put forward by Moore and Wishart and termed the *overnight* test. Five or six test pieces are taken and the Rockwell hardness of each determined.

The specimens are then subjected to about 1 400 000 cycles of stress in a rotating bar machine so as to cover a range of values on both sides of the estimated fatigue limit. After testing in this way the specimens are removed from the machine and pulled in ordinary tension. If a specimen breaks before completing the standard number of cycles its tensile strength for the purpose of plotting is regarded as zero.

The results of the tension tests are corrected to Rockwell hardness and the results plotted against the magnitude of the reversed stress. The resulting curve will, in general, show a maximum or greatest value of the reversed stress which is taken as giving the fatigue limit.

The following results are taken from a test by Moore and Wishart—

Magnitude of Reversed Stress Applied for 1 400 000 cycles lb. per in.^{2}	Rockwell Hardness Number, B Scale	As Tested lb. per in.^{2}	Tensile Strength after 1 400 000 cycles reduced to Rockwell Hardness Scale of 60 lb. per in.^{2}
26 000	70·0	61 500	61 500
28 000	61·2	62 400	61 200
30 000*	60·0	63 200	63 200*
31 000	60·0	62 000	62 000
32 000	60·2	62 000	62 300
33 000	61·2	0†	

* The fatigue limit is thus 30 000 lb. per in.^{2}

† Specimen broke before completing 1 400 000 cycles.

The results of other tests are shown in the graphs in Fig. 157.

The theory on which the test is based is that under repeated stressing materials tend to increase in strength, due presumably to cold work, while on the other hand they suffer loss of strength through fatigue cracks or otherwise. The beneficial influence predominates below the fatigue limit and also to some extent

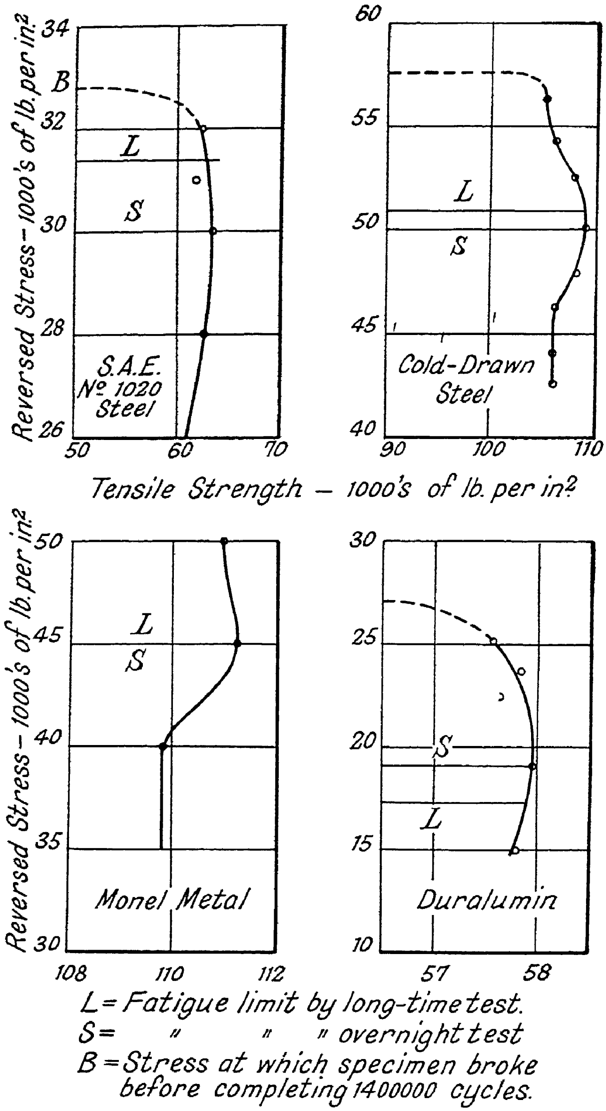


FIG. 157. RESULTS OF FATIGUE TESTS BY THE "OVERNIGHT" METHOD

(*American Society for Testing Materials*)

above that limit, but here fatigue cracking leads to ultimate failure.

Gough's view is that there is no reason why any short-time test should be expected to prove reliable, and that a dividing line at the fatigue limit above and below which material behaves differently does not exist, and further, the variability in commercial material may be sufficient to mask effects brought out by such a test as that proposed above.

The effect of combined bending and torsion on the fatigue properties of materials has also been studied. In this connection a recent paper (*Proc. Inst. Mech. Eng.*, vol. 131, p. 3) giving the results of an investigation at the National Physical Laboratory should be consulted.

CHAPTER XI

ELASTIC CONSTANTS: TESTING OF WIRE AND SHEET METAL

Modulus of Elasticity of Wires. The elastic constants of material in the form of wire and sheet metal are often conveniently determined by indirect methods.

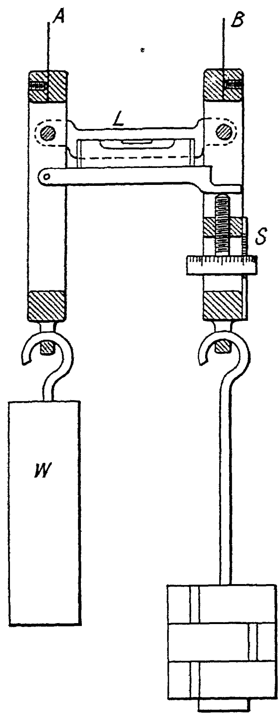


FIG. 158
SEARLE'S APPARATUS FOR DETERMINING YOUNG'S MODULUS OF WIRES

For the direct determination of Young's modulus for a wire, Searle's apparatus is available. (Fig. 158.) Two wires *A*, *B*, eight to ten feet long, are suspended vertically from a beam and at their lower ends carry brass stirrups which support the ends of a sensitive spirit-level L. The level is supported by a pivot in one stirrup and by the point of a micrometer screw in the other. The wire carrying the pivoted end of the level is simply a suspension wire and is kept taut by a weight W attached to the stirrup. The other wire is the test wire. In commencing a test the level is first adjusted to the horizontal so that the bubble is at the centre of its run. A 1 lb. weight is placed on the hanger carried by the test wire and by means of the micrometer screws the level is again brought to the horizontal position. The difference between the micrometer readings gives the extension of the test wire. Readings are taken for 8 or 9 loads and a load-extension curve plotted. The procedure for determining Young's modulus is similar to that described on page 72. The extension of the wire can be measured to an accuracy of 0·01 mm., or approximately to 0·002 mm. by estimation.

Connecting links prevent one stirrup from twisting relatively to the other about a vertical axis but permit relative motion of the stirrups in a vertical direction. The apparatus is made by Messrs. W. G. Pye & Co., Cambridge.

The Amsler Wire Testing Machine. For thick wires some form of extensometer such as the Westinghouse described on page 113 may be used, the load being applied by a wire tester. The Amsler machine is shown in Fig. 159. The upper gripping head is suspended from a lever attached to the inner end of the pendulum spindle. The inclination of the pendulum, which is a measure of the load, is transmitted by a set of parallelogrammatic rods to a pointer moving over a graduated dial. One arm of this parallelogram moves a horizontal rod which engages a small pinion on the axle of the pointer and moves the pointer proportionally to the load.

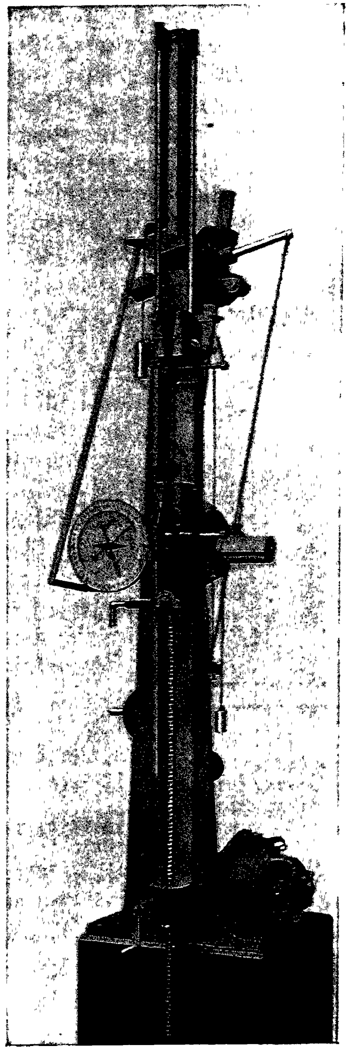


FIG. 159. AMSLER WIRE TESTING MACHINE

(Alfred J. Amsler & Co.)

A 1 000 lb. machine has ranges of 1 000, 500, 200, and 100 lb. The various ranges are obtained by removing or adding weights to the pendulum or by altering the length of the pendulum rod.

To measure the elongation of the specimen use is made of a vertical rod having a $\frac{1}{20}$ in. graduation. If the variation in the distance between the two gripping heads is to be regarded as giving the elongation of the test wire, the rod is fastened to the lower gripping head by a clamp, so that during the test it

If I represents the moment of inertia we have then,

$$\tau = 2\pi\sqrt{\frac{\theta}{T/I}}$$

and substituting for T,

$$\tau = 2\pi\sqrt{(32lI\theta/\pi N\theta d^4)}$$

whence it is seen that θ cancels and the periodic time is independent of the angular displacement. For this to hold, however, the angular displacement and the restoring force must be proportional throughout the range.

If the moment of inertia of the bar is not known it may be calculated or it may be determined as follows.

Two small masses m, m are attached to the ends of the bar equidistant from the axis of the wire. If the masses are of small dimensions their radius of gyration may be taken to be the same as the distance r of the centre of each mass from the axis of the wire. Then, approximately, their combined moment of inertia about this axis is $I_1 = 2\,mr^2$.

The bar is given a slight displacement and the time of an oscillation is found by noting the time for 30 complete oscillations. The time of oscillation is determined similarly when the bar carries the masses m, m.

Since $T_1^2 \propto I$ in the first case, and in the second $T_2^2 \propto I_1 + I$, the ratio of the squares of the periodic times is thus

$$\tau_1^2/\tau_2^2 = I/(I + I_1)$$

Hence $$I = \tau I_1/(\tau_2^2 - \tau_1^2)$$

Having found the moment of inertia of the bar and the periodic time, the modulus of rigidity can be calculated from the formula

$$N = 120lI\pi/d^4\tau^2$$

Instead of a bar a circular disc may be used, with a thin annular ring for the added mass.

The moment of inertia of a rectangular bar of length l, breadth b, and depth d about a vertical axis through its c.g. parallel with the depth is

$$\text{mass} \times [(b^2 + l^2)/12]$$

For a round bar of length l and diameter d,

$$I = \text{mass}(d^2/16 + l^2/12)$$

For a circular disc of diameter d about an axis perpendicular to its plane I = mass $(d^2/8)$.

Searle's Dynamical Method of Determining Young's Modulus. A dynamical method, due to Searle, may be employed to determine Young's Modulus.

The ends of the wire are soldered into two clamping screws which are secured to two equal bars AB and CD, Fig. 161. Two light hooks are screwed into the bars at G and G' so that the hooks are perpendicular to the wire. The system is suspended by two parallel threads at least 20 in. long attached to the hooks. If the bars are turned through equal angles ϕ in opposite directions and then set free the system will execute harmonic vibrations in the horizontal plane. If the vibrations are small the effects of the horizontal and vertical displacements of the centres of the bars may be neglected.

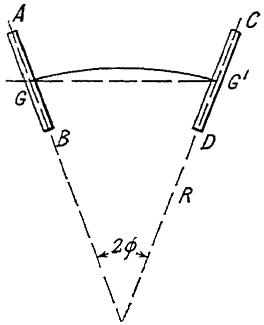


FIG. 161. ILLUSTRATING METHOD OF DETERMINING YOUNG'S MODULUS BY BENDING OSCILLATIONS OF A WIRE

For all practical purposes the forces on the system reduce to equal couples exerted by the bars on the wire which is bent to the arc of a circle.

The bending moment is EI/R where E is Young's modulus, I the moment of inertia of the section of the wire about an axis through its centroid and R the radius of curvature to which the wire of length l is bent. $I = \pi d^4/64$ in.^{4} and from the figure $R = l/2\phi$.

If I_D is the moment of inertia (dynamical) of either bar about a vertical axis the angular acceleration of the bar is

$$\alpha = \frac{\text{couple}}{\text{moment of inertia}} = \frac{EI}{I_D R} = \frac{2EI\phi}{I_D l}$$

and the period of oscillation

$$\tau_E = 2\pi\sqrt{(I_D l/2EI)}$$

EXAMPLE: Consider a steel wire 0·047 in. diameter and 15 in. long. Moment of inertia of the bar $I_D = 13$ lb.-in.^{2} units. Average time for one complete oscillation 1·188 sec. The value of E is required in lb. per in.^{2} so $g = 32{\cdot}2 \times 12$ in. per sec. per sec.

Hence $$I_D = \frac{13}{32{\cdot}2 \times 12}$$

and $$E = \frac{128 \times 13 \times 15 \times \pi}{(1{\cdot}188)^2 \times 32{\cdot}2 \times 12 \times (0{\cdot}047)^4}$$

$$= 29{\cdot}65 \times 10\cdot \text{ lb. per in.}^2$$

In carrying out this experiment the amplitude of vibration must be small; not much greater than 3°. The ends of the two bars should be tied together in the constrained position by a piece of thread and vibration started by burning the thread. A pointer should be set close to the end of one of the bars to assist in finding the time of vibration. Several measurements of the diameter should be made at various points along the wire.

The modulus of rigidity may be found as already described, page 204, by clamping one bar to a support and noting the time of oscillation of the suspended bar. Then

$$N = 128\pi I_D L / \tau_N^2 d^4$$

The ratio of E to N is simply

$$E/N = \tau_N^2 / \tau_E^2$$

which allows Poisson's Ratio σ to be calculated (page 14.)

$$\sigma = E/2N - 1$$

Values of the elastic constants of several materials in the form of wire are given in the following table—

TABLE XVI

ELASTIC CONSTANTS OF WIRES

(G. F. C. Searle)

Material	E lb. per in.^{2}	N lb. per in.^{2}	σ
Carbon steel	$28{\cdot}72 \times 10^6$	$11{\cdot}41 \times 10^6$	0·258
Brass (hard drawn)	14·81	5·392	0·376
Phosphor bronze	17·40	6·320	0·378
Copper (annealed)	18·73	4·083	0·608
Nickel (hardened)	34·78	10·76	0·614
*Platinoid	19·70	5·217	0·887
†German silver	16·72	3·795	1·207

* 54 Cu. 24 Ni. † Cu. Ni. Zn. Alloy.

From the relation $3K(1 - 2\sigma) = 2N(1 + \sigma)$ we see that if $\sigma > \frac{1}{2}$ either K or N would be negative. The explanation is that the material of the last four wires is not isotropic.

Searle's Method of Determining the Elastic Constants of Strip Metal. Metal in the form of strip can be tested in tension and the value of Young's modulus found in the usual way. The modulus may also be found by loading a thin strip of metal as a beam. The simple theory of bending then fails and Poisson's ratio enters into the relation between deflection and bending

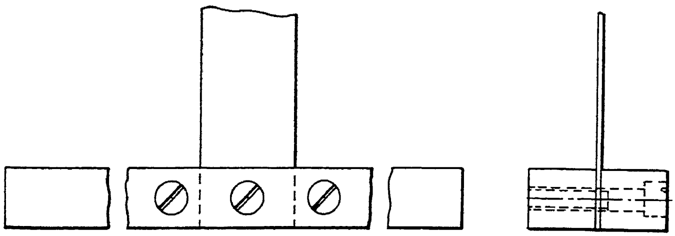


FIG. 162. INERTIA BARS CLAMPED TO STRIP

moment. Because of this the formula connecting Young's modulus and the bending moment is not, in itself, sufficient for the determination of E from a series of observations of load and deflection, and it is necessary, in addition, to find the modulus of rigidity by an oscillation method.

A convenient size of specimen for the purpose is a strip about 24 in. long × 2 in. wide. A hole is drilled at each end and the strip firmly clamped between bars as shown in Fig. 162. One pair of bars is fixed firmly to a suitable support so that the strip hangs vertically and the lower bars are allowed to oscillate in a vertical plane. The procedure is similar to that described on page 204.

If l be the length of the strip,
$2a$ the width of the strip,
$2b$ the thickness of the strip,

the relation between the angle of twist and the twisting couple is

$$T = 16Nat^3\phi/3l$$

If L is the length of each bar, A its width and M its mass the moment of inertia of the two bars about the axis of the strip is, by the theorem of parallel axes,

$$I_D = \tfrac{2}{3}M(L^2/4 + A^2) + 2M(A/2 + t)^2$$

Neglecting the second term we have approximately the value

$$I_D = \tfrac{2}{3}M(L^2/4 + A^2)$$

The angular acceleration is T/I_D, that is $16Nat^3\phi/3lI_D$.

Hence if τ be the periodic time

$$N = 3lI_D\pi^2/4at^3\tau^2 \text{ lb. per in.}^2$$

The relation between E and the bending moment M required

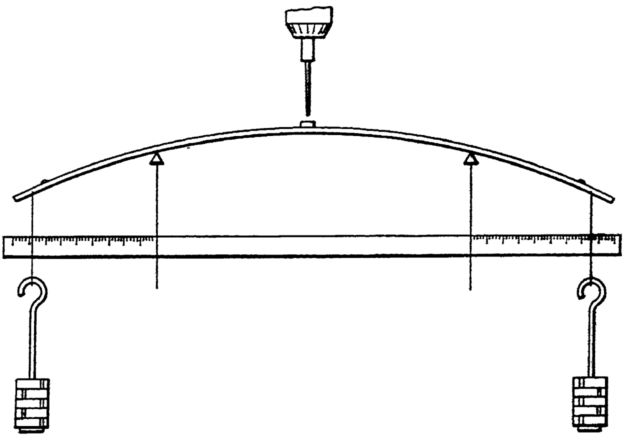


FIG. 163. METHOD OF SUPPORTING AND LOADING STRIP DURING DEFLECTION TEST

to bend the strip so that the longitudinal filaments have a radius of curvature R is

$$M = 4at^3E/3(1 - \sigma^2)R$$

In performing the bending test the strip is supported on knife-edges, Fig. 163, which should be spaced so that the part of the blade between them is as nearly as possible straight when unloaded. This distance is approximately half the length of the blade. The deflections may be measured by means of a pointed micrometer forming part of an electric circuit arranged to light a small bulb when contact occurs. Or the curvature may be determined by the mirror method to be described later.

Small hangers are suspended by threads attached to the ends of the beam with seccotine. Readings are taken with several

loads on the hangers. The radius of curvature is calculated from the formula

$$R = l^2/2h + h/2$$

where d is the distance between the supports and h the deflection of the central point.

If W is the load on each scale pan and p the average distance of the hangers from the supports

$$E/(1 - \sigma^2) = 3WRp/4at^3$$

The mean value of $E/(1 - \sigma^2)$ is obtained by plotting a graph.

Substituting for E from $E = 2N(1 + \sigma)$

we have

$$2N(1 + \sigma)/(1 - \sigma^2) = 3WRp/4at^3$$

therefore

$$\sigma = 1 - 8Nat^3/3WRp$$

Substituting this value of σ in

$$E/2N = (1 + \sigma)$$

we have

$$E = 4N(1 - 4Nat^3/3WRp)$$

The theory of the foregoing method is given fully in Searle's *Experimental Elasticity.*

Determination of Poisson's Ratio by the Method of Flexures. The change of slope of a bent beam is best observed by means

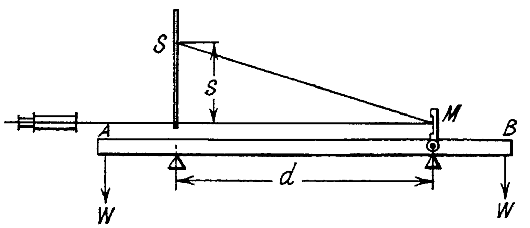


FIG. 164. METHOD OF MEASURING THE SLOPE OF A BEAM

of a telescope and scale. A plane mirror M is attached to the beam immediately over a knife-edge, and a scale S and a telescope are arranged as in Fig. 164, the ray of light from the mirror to the telescope being horizontal and the scale vertically over the knife-edge support.

If d is the distance between the knife-edge supports and if

the tangent to the beam at the knife-edge turn through a small angle and give a displacement on the scale of amount s for a load W applied at A and B then the radius of the arc to which the beam is bent is

$$R = d^2/s$$

If h is the deflection of the mid point of the beam we have from the geometry of a circle $R = d^2/8h$. Comparison with the previous result shows that the movement over the scale is eight times the deflection of the beam.

Greater accuracy is obtained if the distance between the

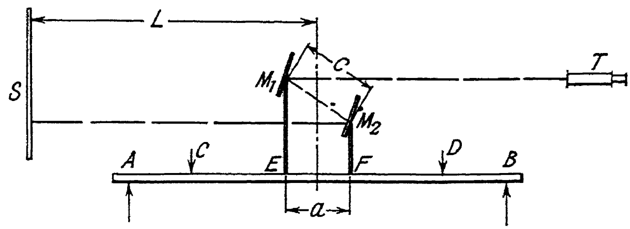


FIG. 165. CARRINGTON'S METHOD OF MEASURING FLEXURE

scale and the mirror be increased and the curvature calculated from the change of slope.

Carrington's method of determining E and σ by the method of flexures is as follows—

The beam is supported at A and B as in Fig. 165 and loaded at C and D. Two mirrors M_1 and M_2 are attached to small columns fixed to the beam at E and F. The distance EF in Carrington's experiments was 1 in. Observations being taken by means of a telescope and scale, the modulus of elasticity may be calculated from the formula

$$E = \frac{24a(L + c/2 + a/2)}{bt^3} \cdot \frac{M}{s}$$

where

b is the breadth of the beam,
t the thickness,
a the distance between the mirror columns,
c the distance between the mirrors, and
L the scale distance.

To determine Poisson's Ratio the lateral curvature must be

measured. The mirrors are arranged as in Fig. 166. If the dimensions L, a and c are the same as in the previous experiment it is only necessary to plot curves connecting bending moment and scale reading for both cases, when the ratio of the slopes of the resulting straight-line graphs will be the value of Poisson's Ratio.

In any case Poisson's Ratio is given by

$$\frac{\text{Longitudinal curvature}}{\text{Lateral curvature}}$$

The thickness of the beam must not be too great to prevent a measurable degree of lateral curvature under moderate loads.

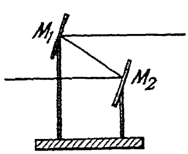


FIG. 166. CARRINGTON'S METHOD OF MEASURING LATERAL FLEXURE

Poisson's Ratio may be found by measurement of the longitudinal and lateral strains in a test bar under tension, the lateral strains being measured by means of a delicate extensometer, as for instance Coker's extensometer described on page 105.

In a recent investigation, Lyse and Godfrey employed a split collar of spring steel clamped on to a tension specimen by set screws and giving a magnification of approximately two at the gap. The amount of gap opening was measured by means of a Huggenberger extensometer placed across the gap. Lateral strains were measurable to 0·000005 in. for a 1 in. diameter specimen.

These investigators determined Poisson's Ratio by several methods and found that for structural steel σ varied from 0·271 to 0·302, while for alloy steels the ratio varied between 0·272 and 0·320.

Commercial Tests on Wire. Commercial tests on wire comprise tensile, bending, twisting and wrapping tests. Tensile tests are conveniently made in a machine of the type described on page 203. Some British specifications require that a lever type machine be used. The sample of wire must not be straightened or in any way prepared before testing.

The specified rate of application of the load varies with the material being tested. Nine-tenths of the minimum breaking load must be applied quickly and the remainder at a steady rate until fracture of the wire occurs. With galvanized stay wire, for instance, the time to be occupied in applying the final

tenth of the load is given as 20 sec., the total time from the application of the load to the break being approximately 30 sec.

Twisting tests are made by clamping the ends of the test wire in suitable grips, one of which can be rotated, while the other, although prevented from rotating, is free to slide longitudinally. (See Fig. 96.) Small hand machines are usually employed for this test. The twist is applied through a geared handwheel and a counter registers the total number of turns up to fracture.

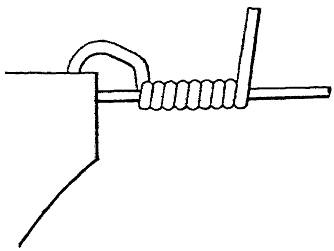


FIG. 167. WRAPPING TEST

Specifications usually call for a minimum number of twists in a given length without fracture, the number of twists to be determined by first making an ink mark on the untwisted wire and then counting the number of spirals in the gauge length. The full number of twists must be visible between the grips. The speed of testing should not exceed one revolution per second.

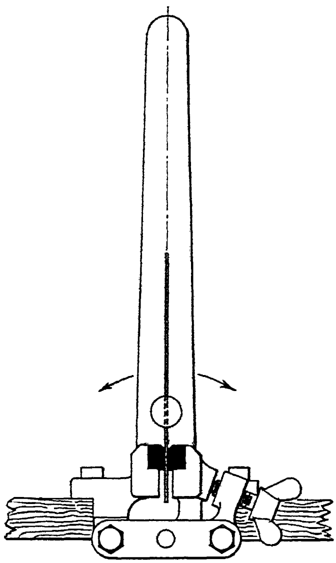


FIG. 168. HAND BENDING MACHINE FOR WIRES

For hard drawn copper wire the specified number of twists ranges from 15 to 30 on a length of 6 in., the number depending on the diameter of the wire.

The wrapping test is made by gripping the wire in a vice and wrapping one portion of the wire around the other as shown in Fig. 167.

The wire is wrapped six times around its own diameter in the same direction, unwrapped, and again wrapped in six turns in the same direction as the first wrapping. In some cases only one wrapping is specified.

Bending tests are made by means of an apparatus shown in Fig. 168, the wire being bent to and fro through 90° or 180° over a specified radius. For phosphor bronze wire the angle of

kept rotating until it eventually breaks by fatigue. Fracture occurs at or near mid-span and not near the chuck.

The wire does not "whirl" about the straight line AB but rotates about its own curved axis of flexure. The speed of operation is about one million cycles per hour.

When the angle of flexure is small the theory of the test corresponds to that of the Euler strut (page 21). The curve assumed by the centre line of the wire is approximately

$$y = Y \sin (\pi x/\mathrm{L})$$

where

Y = deflection at the mid-point,

L = length of sample,

y = deflection at a distance x from one end.

The inclination and curvature are given by

$$i = dy/dx = (\pi Y/\mathrm{L})\cos(\pi x/\mathrm{L}) = \theta \cos (\pi x/\mathrm{L})$$

$$1/r = d^2y/dx^2 = -(\pi^2 Y/\mathrm{L}^2) \sin (\pi x/\mathrm{L})$$

$$= -(\pi\theta/\mathrm{L}) \sin (\pi x/\mathrm{L})$$

θ is the inclination at the end, as indicated by a vernier on the swinging headstock of the machine.

The greatest curvature is at the mid-point and is given by

$$1/\mathrm{R} = \pi\theta\mathrm{L}$$

where R denotes the corresponding least radius of curvature.

The bending strain at the mid-point of a wire of diameter d is

$$e = d/2\mathrm{R} = (\pi/2)\theta(d/\mathrm{L})$$

and the bending stress

$$f = \mathrm{E}e = (\pi/2)\theta(d/\mathrm{L})\mathrm{E}$$

where E is Young's modulus.

When the angle θ is greater than about 20° the curve of flexure becomes the "Elastica" and the foregoing results need slight correction.

In addition to the bending stress a small compressive stress acts in the wire. This may be calculated by using Euler's formula, but generally the compressive stress may be ignored.

The end thrust applied to the test piece and the grip of the

chuck affect the whirling critical speeds. These several speeds —which are to be avoided—are proportional to

$$\frac{\sqrt{(E/\rho)}}{d(L/d)^2}$$

where ρ is the density of the metal. In general it is desirable to run between the second and third criticals.

Test pieces are usually 150 diameters long and lengths of from $3\frac{1}{2}$ to 30 in. may be tested.

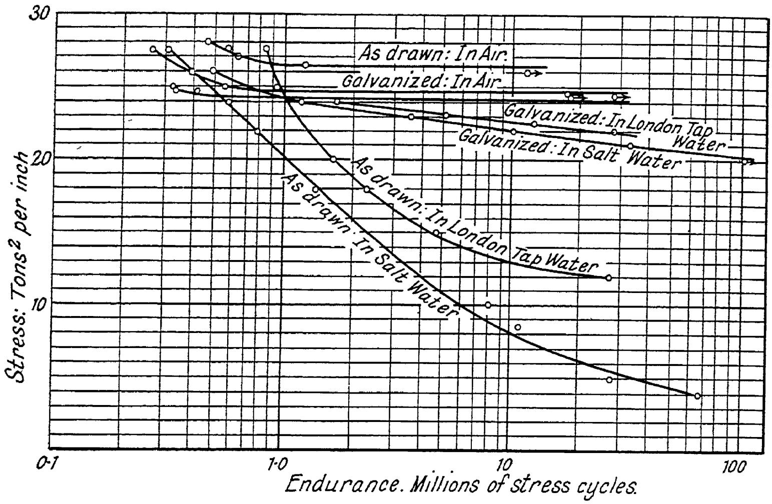


FIG. 170. ENDURANCE CURVES FOR STEEL WIRES UNDER CONDITIONS OF CORROSION FATIGUE
(Bruntons (Musselburgh) Ltd.)

Some endurance curves for steel wire under various conditions of corrosion fatigue are given in Fig. 170.

Tests of Sheet Metal. The mechanical tests made on sheet metal comprise tensile tests, bend tests, reverse bend tests, hardness tests, and a cupping test.

TENSILE TESTS. Various forms of test piece are called for in specifications. Two forms only have been standardized by the British Standards Institution for material not exceeding 0·128 in. (10 S.W.G.) in thickness.

The "short" form has a gauge length of 2 in., is $\frac{1}{2}$ in. wide, and has a parallel length of $2\frac{1}{4}$ in. The "long" form, intended to be used in cases where the general elongation of the test piece

is likely to be more informative than the local elongation, has a gauge length of 8 in., is $\frac{3}{4}$ in. wide, and has a parallel length of 9 in.

The radius of the transition curve at the ends of the straight portion is 1 in. The preparation of the test piece is somewhat difficult especially from very thin sheets. A combined filing and drilling jig in which the specimen can be dressed to shape is of great assistance, as axial loading of the test piece is essential.

For measuring extensions the extensometer employed should possess a sensitivity of 1/20 000 in. For elastic measurements a

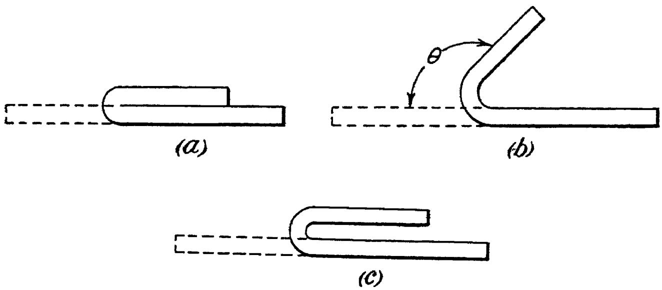


FIG. 171.* BEND TESTS FOR SHEET METAL

form of mirror extensometer may be used, or a pair of Huggenberger extensometers, clamped one on each side of the specimen. If measurements are to be carried into the plastic region a coarser form of instrument must be employed. A form of direct-reading instrument has been designed to meet the necessary requirements, and a diagram of this is given in the B.S.S. No. 485.

With some materials the speed of testing is not without influence on the result, but so far it has been found impossible to specify generally rates of testing.

BEND TESTS. In bend tests the bending is caused by pressure or by a succession of blows. The metal is bent as in Fig. 171, (*a*), (*b*), and (*c*), being closed on itself or bent through a specified angle. The test as in (*a*) is known as the *hammer* or *seaming* test. Tinplates usually stand this test when the line of bend is "across the grain," i.e. at right angles to the direction of rolling,

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but not infrequently fail when bent "with the grain,' i.e. parallel with the direction of rolling.

Considerable differences of opinion exist as to the value and fairness of this test.

The test is sometimes made less severe by bending the sheet through an angle round a specified radius as in (*b*). The angle may reach 180° as in (*c*).

The test piece must remain in contact with the former during the bending operation and if, after removing the constraint, the test piece assumes a slightly different shape this is to be ignored. Specimens are deemed to be satisfactory if no cracks show on the convex side.

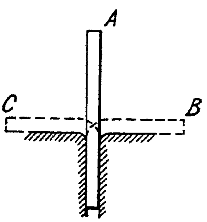

FIG. 172.* REVERSE BEND TEST

REVERSE BEND TESTS. A strip of the material *A*, Fig. 172, is clamped in a vice provided with rounded jaws and is then bent

(*a*) through 90° into position *B*, then back to *A* and then to and fro between these positions; or

(*b*) is first bent into position *B*, then through 180° into position *C* and then alternately into positions *B* and *C* until a crack appears.

The number of bends to fracture is counted, the first bend through 90° being ignored, although in the tinplate trade it is counted as half a bend.

Some constraint is needed to ensure contact between the test piece and the jaws of the vice. If this condition be not fulfilled the radius to which the strip bends will generally differ appreciably from the radius of the jaw. In the Jenkins Bend Tester, an improved form of the machine shown in Fig. 168, a strip about 2 in. long and ½ in. wide is gripped between jaws which are rounded to a radius of 0·04 in. This radius is used for all sheets up to 0·4 mm. For sheets up to 2 mm. a larger machine is used with jaws of 0·08 in., 0·125 in. and 0·25 in. radius. A hand-lever working on a floating fulcrum is provided with a hardened steel roller which catches the projecting strip and permits it to be bent to and fro. Pressure is applied to the roller

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by means of a spring so that in the course of making a bend the test piece is forced to conform with the curvature of the jaw.

The length and width of the strip appear to be without influence on the results provided the strips are cut from the sheet with a close-set sharp pair of shears; otherwise the burred edges tend to crack prematurely.

With a given material the number of bends which a test

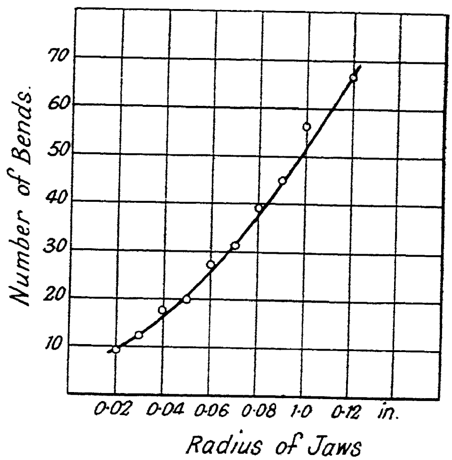


FIG. 173. INFLUENCE OF RADIUS OF BEND ON NUMBER OF BENDS TO CAUSE FRACTURE
(*J. G. Godsell*)

piece will withstand will depend on the direction in which it has been cut from the sheet.

(See Bibliography 261.)

It has been established that if

n_0 = the number of bends sustained by a strip taken from from the direction of rolling.

n_{90} = the number of bends sustained by a strip perpendicular to the direction of rolling,

the number of bends n_θ that a strip cut at an angle θ to the direction of rolling will withstand is given by

$$n_\theta = \sqrt{(n_0^2 \cos^2 \theta + n_{90}^2 \sin^2 \theta)}$$

In the application of this formula the initial bend through 90° is counted as half a bend.

With mild steel sheets the rate of bending appears not to affect the result, but whether this holds for all alloys has not been definitely established.

The influence of the radius of the bend is shown in Fig. 173 which is plotted from some tests of annealed mild steel sheets

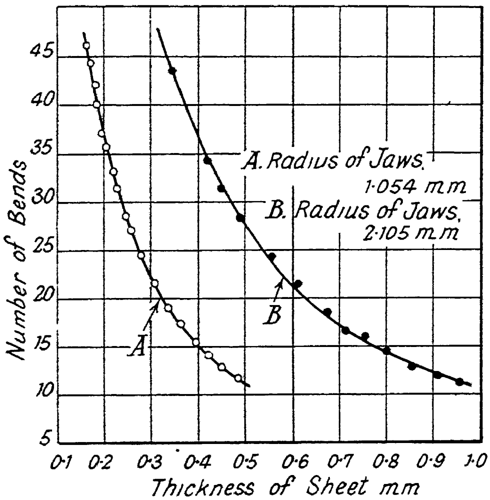


FIG. 174. INFLUENCE OF THICKNESS OF SHEET ON NUMBER OF BENDS TO CAUSE FRACTURE

(*J. G. Godsell*)

0·34 mm. thick. Each plotted point represents the mean of 10 tests, the strips being bent through 180° and the first 90° bend counted as half a bend. The effect of the thickness of the sheet with a given radius of jaw is shown in Fig. 174.

If the number of bends be plotted against the ratio

$$\frac{\text{Radius of bend}}{\text{Thickness of strip}} = \frac{\mathrm{R}}{\mathrm{T}}$$

a straight line is obtained having the equation

$$\mathrm{N} = \mathrm{K}(\mathrm{R/T} - 0{\cdot}8)$$

in which K depends on the material.

Logarithmic plotting leads to the result $\mathrm{N} = \mathrm{C}(\mathrm{R/T})^{\frac{5}{4}}$ for

mild steel, where C depends only on the quality of the material. Fig. 175.

TABLE XVII

STANDARD FIGURES FOR TINPLATE AND BLACKPLATE
(For use with the Jenkins Alternating Bend Tester (0·040 Jaws)

NUMBER OF BENDS REQUIRED TO FRACTURE

Thickness		*Substance	Ordinary Coke		Deep Stampers	
			Weak Way	Strong Way	Weak Way	Strong Way
in.	mm.					
0·0155	0·395	135	4½	7½	5	8½
0·0146	0·371	127	5½	9½	6	10½
0·0124	0·315	108	7	12	8	13½
0·0116	0·295	100	7½	13	8½	15
0·0109	0·277	95	7½	13	8½	15½
0·0104	0·264	90	8½	15	9½	16½
0·0098	0·248	85	9½	16½	11	19
0·0092	0·234	80	10½	18	—	—
0·0086	0·218	75	11½	20	—	—
0·0081	0·206	70	12½	22	—	—
0·0074	0·188	65	13½	24	—	—

CUPPING TEST. The object of the cupping test is to ascertain the ductility of the material. The test piece, which is a disc about 3 in. diameter cut from the sheet, is placed between two steel rings of rectangular section and enclosed in a box or frame. A punch having a rounded nose is placed in contact with the disc and is forced down by means of a screw and nut or by compression in a small testing machine. The load, and the depth of the cup when fracture is observed, provide a measure of the ductility of the metal.

In Continental practice the disc is gripped firmly between the rings, but in this country and America clearance is allowed so that the test piece can draw down with but little restriction. In the form of test known as the *Erichsen Test* the internal diameter of the ring is 27 mm. and the radius of the nose of the punch 10 mm. Practice differs slightly in different countries.

The standard Erichsen machine is shown in Fig. 176. In

* Weight in lb. of a standard box of tinplates, i.e. 112 sheets 14 in. × 20 in. = 31 360 in.^{2}

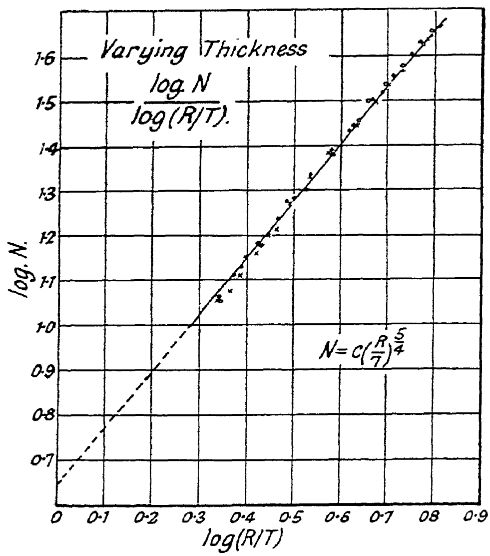


FIG. 175. RELATION BETWEEN NUMBER OF BENDS TO FRACTURE AND THE RATIO RADIUS OF BEND-THICKNESS OF SHEET
(*J. G. Godsell*)

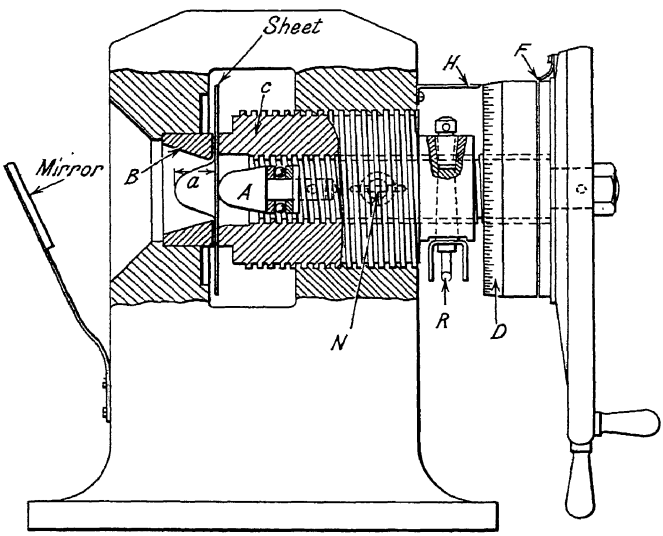


FIG. 176. STANDARD ERICHSEN MACHINE FOR CUPPING TEST

making the test the scale *D* is set to zero by shifting the movable collar on which it is engraved until the spring *F* snaps into a small hole in the collar. The specimen is inserted between the die *B* and the holder *c* and the handwheel turned until the specimen is firmly clamped. The thickness can then be read off from the scale *D*. The total range of this scale is 5 mm. and readings can be continued on scale *H*.

After the thickness of the sheet has been noted the handwheel is turned back five divisions on scale *D* ($\frac{5}{100}$ mm.) in order to give the test piece a small amount of play. The holder *c* is secured in this position by means of the wing nut *N*. The scale on *D* is now moved until its zero coincides with that of *H*. The gear is changed by pressing against the milled ring *R* and the handwheel turned clockwise. The tool *A* now moves forwards and bulging of the sheet is noted in the mirror. The image in the mirror is carefully watched until fracture occurs, when the depth *a* of the impression is read off on the scale. The rate of testing should be reduced as the point of fracture is approached, in order to obtain an exact reading.

This type of test seems to be regarded as a method of detecting unsuitable material, but one that is less discriminating than the tensile or alternating bend tests where more satisfactory material is being dealt with. The test certainly provides valuable information regarding the probable surface appearance of a finished pressing, this being related to the grain size of the material.

In one respect the test appears to be superior to the tensile test, in that it tests the ductility of the sheet in all directions. Factors disadvantageous to the test from a quantitative aspect are the uncertainty of the frictional effects at the surface in contact with the punch, the amount of drawing at the grips, and the determination of the exact point at which fracture commences.

However, the cupping test is largely used in the routine testing of metals, and some curves of Erichsen values are given in Figs. 177A, 177B, 177C, and 177D. The curves, with one or two exceptions, are not the Erichsen Standard Trade Quality Curves.

The depth of cup at fracture will not itself provide a true index of the deep drawing qualities of a material, but consideration must be given to the type of fracture and the appearance of the dome. The fracture should be circumferential in metals which are required for drawing operations. Metal which

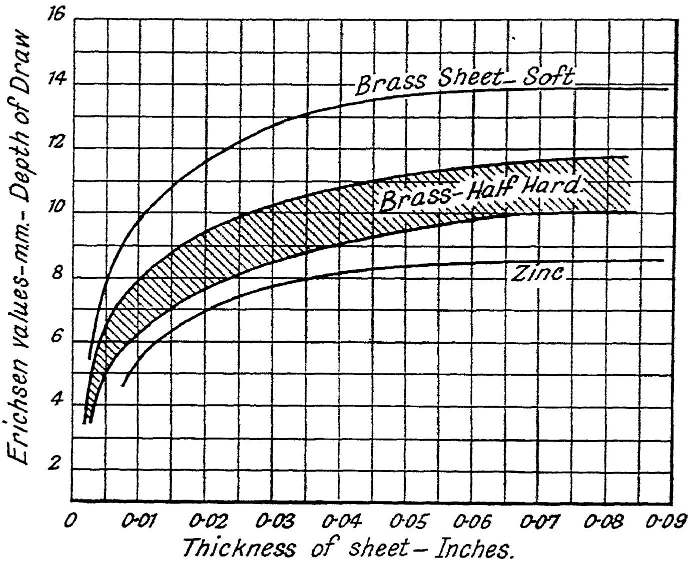


FIG. 177C. CURVES OF ERICHSEN VALUES

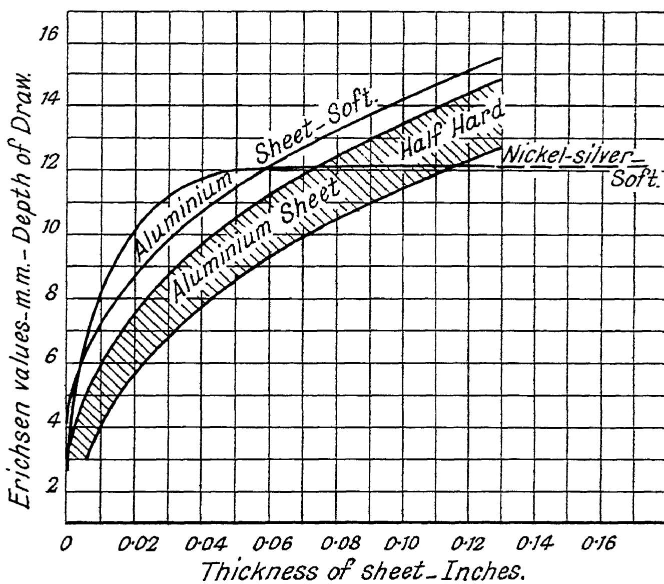
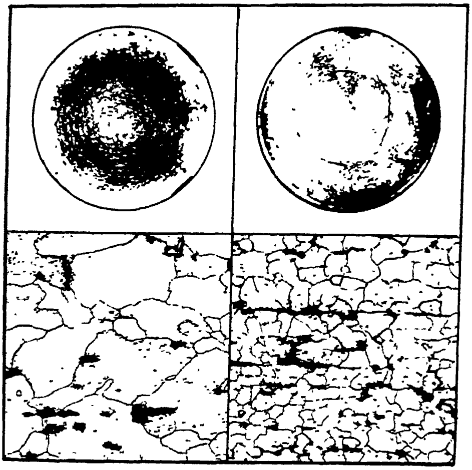


FIG. 177D. CURVES OF ERICHSEN VALUES

has been reduced considerably by cold rolling, such as hard nickel-silver, will give a fracture in one direction only and will not be suitable for folding and drawing. A rough or crinkled dome indicates a loose or coarse structure that will offer increased resistance to the drawing tools, and may result in premature breakage even though the metal is soft.

With non-ferrous metals and alloys, Armco iron, and electro-deposited iron, the roughness is invariably due to over-annealing, but this is not true in the case of dead mild steel sheet.



(a) (b)

FIG. 178. APPEARANCE OF DOME IN MILD STEEL

(a) Rough dome—coarse grained structure.
(b) Smooth dome—same material commercially re-annealed.
(*J. G. Godsell*)

Abnormal grain growth, and therefore a rough Erichsen dome, in the case of dead mild steel sheet, is due to under-annealing after cold work. On annealing above Ac_3, i.e. 900° C., the structure, no matter how coarse, is refined and is not appreciably coarsened at any temperature below 1 200° C. Good normalized mild steel exhibits a smooth dome (Fig. 178). A dome close grained in appearance is usually only encountered in non-ferrous materials and is generally caused by oxidation of the metal during annealing or by excessive pickling. In all cases the domes should be smooth, and deep-drawing qualities should not possess a value falling below the respective curves shown. Half-hard materials should fall between the curves.

Hardness Values for Various Sheet Materials. The following values give tensile and elongation values for brass and hardness values for several materials.

TABLE XVIII

VICKERS PYRAMID NUMERALS

Load: 5 kg.

Grade	Bright Mild Steel		Deep Drawing Mild Steel		Welsh Plate		
	Min.	Max.	Min.	Max.	Quality	Min.	Max.
1. Soft	—	110	—	105	P.C.A.	80	110
2. Medium hard	127	156	—	—	C.R.C.A.	100	120
3. Hard	170	—	—	—	P.C.R.C.A.	95	120

TABLE XIX

TENSILE AND ELONGATION VALUES FOR BRASS OF VARIOUS TEMPERS

	Tons per in.^{2} Minimum	Elongation Percentage on 2 in.
1. Soft	18	46
2. Half hard	24	26
3. Hard	34	8
4. Spring	40	4

TABLE XX

VICKERS PYRAMID NUMERALS

Load: 5 kg.

	Brass		18% Nickel Silver		Phosphor Bronze		Aluminium		Sheet Copper	
	Min.	Max.	Min.	Max.	Min.	Max.	Min.	Max.	Min.	Max.
1. Soft	—	75	—	100	—	100	—	30	—	70
2. Half hard	95	125	130	165	141	185	30	40	75	120
3. Hard	140	155	170	195	190	228	40	55	125	180
4. Spring	165	—	200	—	235	—	—	—	—	—

The Jovignot Test. A method of testing sheet metal has been introduced recently by Jovignot in which a clamped circular

plate, the test piece, is subjected to fluid pressure. Rupture occurs eventually as a result of the pressure and deformation. It is found that the sheet deforms to an approximately spherical segment and that the stress at fracture can be calculated by the formula for the strength of a spherical shell, namely

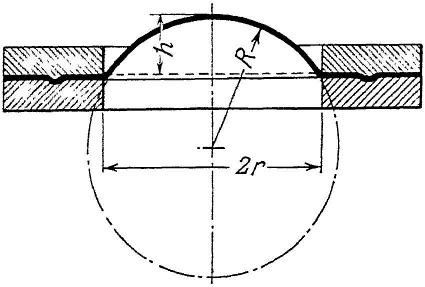


FIG. 179. JOVIGNOT TEST

$$\text{stress} = PR/2t$$

where P is the pressure, t the thickness of the sheet and R the radius of curvature $= (r^2 + h^2)2h$. See Fig. 179.

A measure of the ductility can be deduced from the initial and final dimensions of the test piece. The *cupping coefficient*, which is equivalent to the average increase of surface area of the test piece per unit area, is expressed as the ratio h^2/r^2.

The test is now being investigated with a view to its possible standardization.

Rolland and Sorin's Method of Determining Young's Modulus. A novel method of determining Young's modulus has been devised by Rolland and Sorin. The test piece S Fig. 180, of rectangular section, is held firmly in a vertical position by a clamp at its lower end. The upper end carries a platform A on which bear two identical pendulums P and P'. If P be at rest and P' be set in oscillation the slight reactions of the swinging pendulum produce small elastic deformations in the test piece which in turn cause P to oscillate. Eventually the energy of P' is transferred to P, the amplitude of whose oscillations reach a maximum value while the amplitude of the oscillations of P is reduced to zero. Energy is then transferred in the opposite direction until P has been brought to rest and P' again has its largest swing. The cycle is repeated until the energy of the pendulums is finally damped out.

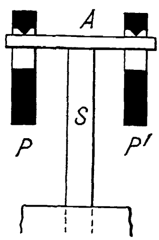


FIG. 180. PENDULUM METHOD OF DETERMINING YOUNG'S MODULUS

The quantity to be observed is the time t which elapses between two successive arrests of the same pendulum. If the

section of the test piece has a breadth b and a depth d in the plane of bending and the length from the point of fixing to the plane of the knife-edges is l, the rigidity K of the bar is the static end deflection for unit load and is given by $K = 3EI/l^3 = Ebd^3/4l^3$ where I is the moment of inertia of the cross-section. The value K determines the amplitude of the movement of the end of the test piece caused by the reaction of the pendulum, and hence the value of t.

If each pendulum has a mass M and time-period T the relation between K and t is

$$K = (2\pi^2 M/T^2)(t/T + 2\mu/M)$$

in which μ is the equivalent mass of the test piece. For a test piece of mass m fixed in the manner shown, the value of μ is $33/140\ m$.

As t is generally large compared with T and the pendulums are heavy so that $2\mu/m$ is negligible,

$$K = 2\pi^2 Mt/T^3$$

and hence

$$E = 8\pi^2 ML^3 t/T^3 bd^3$$

The chief advantages claimed for the method are that measurements of load and extension are replaced by a single time measurement; the test piece has time to settle down under alternations of loading and give a true modulus; and that the modulus obtained is that at the origin of the stress-strain curve.

The following table compares values of E for several materials obtained by the method described and by the usual tension method using a Marten's extensometer.

TABLE XXI

Comparison of Values of Young's Modulus (E) Determined by the Pendulum Method and by the Direct Method Using a Marten's Extensometer

Metal	E Pendulum Method kg. per mm.^{2}	E Marten's Extensometer kg. per mm.^{2}
Steel	21 000	20 830
Duralumin	7 640	7 460
Aluminium	7 650	7 600
Bronze 1	12 000	11 990
Bronze 2	12 500	12 900

CHAPTER XII

SOME TEST PHENOMENA AND RESULTS

Influence of Form of Test Piece on the Shape of the Stress-strain Diagrams. The shape of the load-extension or stress-strain diagram obtained in a tensile test will vary not only for different metals but also for the same material to an extent depending on the chemical composition; on the mechanical and thermal treatment the material has hitherto received; on the form of the test piece; and on the sensitiveness of the testing machine employed.

The influence of the shape of the test piece is brought out in the diagram Fig. 181. The graphs *a* and *b* represent the load-

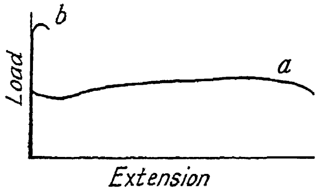


FIG. 181. INFLUENCE OF SHAPE OF TEST PIECE ON THE LOAD EXTENSION DIAGRAM

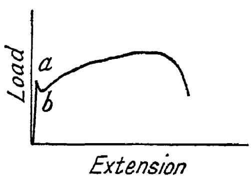


FIG. 182. ILLUSTRATING THE "DROP" AT THE YIELD POINT

extension diagrams obtained with mild steel test pieces 0·6 in. diameter and 3 in. and $\frac{1}{8}$ in. respectively between shoulders.

It will be noticed that the shorter specimen sustained approximately 60 per cent greater load than that carried by the 3 in. specimen and that the drop at the yield point has vanished. The diagrams suggest materials possessing entirely different properties. This example emphasizes the necessity for standardization of test pieces in order to obtain comparable results.

An increase in the carbon content of a steel will likewise cause a marked change in the stress strain-diagram, a high percentage of carbon resulting in little elongation of the test piece and giving a graph similar to *b* of Fig. 181.

The characteristic yield point given by mild steel and wrought iron, at which the specimen elongates without increase of load, or rather, usually with a drop in load, is absent in all other materials. The two points *a* and *b*, Fig. 182, represent

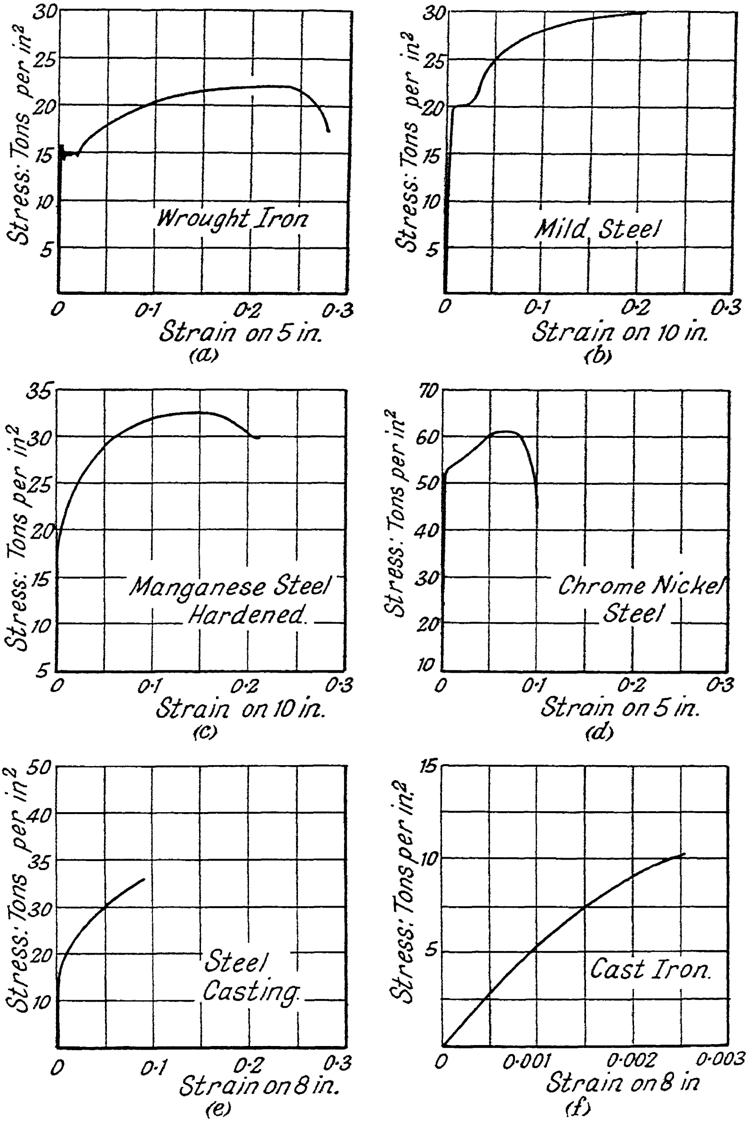


FIG. 183. STRESS-STRAIN DIAGRAM GIVEN BY FERROUS MATERIALS

what are termed the "upper" and "lower" yield points respectively.

The drop in load at the yield point, which is a few per cent in a diagram obtained in a tensile test carried out with the aid of an autographic recorder, has been shown by Dalby, using an optical device (page 114), to amount to 13 per cent, while Cook and Robertson, who employed a special arrangement to overcome the inertia of the elastic system, obtained a reduction of 27 per cent.

The upper yield point is affected by the speed of testing.

Stress-strain Diagrams for Metals in Tension. Various tensile stress-strain diagrams are shown in Fig. 183. (*a* to *f*). The forms (*a*) and (*b*) given by wrought iron and mild steel respectively are very similar. Whether or not a drop is indicated at the yield point will depend on the sensitiveness of the machine and the recording apparatus.

Diagram (*c*) is a curve for hard manganese steel. In the annealed state of the material the stress-strain curve is similar to (*b*) for mild steel. High tensile alloy steels show less and less extension as the tensile strength rises. A curve obtained by Dalby with a 60 ton chrome nickel steel is shown in (*d*). Curve (*e*) is from a steel casting in the "as received" condition. Cast iron (*f*) gives a curve no part of which is straight. Young's modulus is then usually determined as the tangent modulus at the origin.

Stress-strain diagrams for several non-ferrous metals and alloys are given in Fig. 184 (*a* to *f*). Copper shows no definite yield point. A yield point must therefore be determined in an arbitrary manner. It is customary to draw a line parallel to the tangent to the curve at the origin at a point on the strain axis representing a strain of 0·15 or 0·2 per cent. The point of intersection of the line so drawn with the curve is taken to be the yield point.

The curve for aluminium bronze, Fig. 184 (*c*), shows an abrupt change at the yield point.

Tin and zinc show a striking difference from other metals in that the yield point and the point of maximum load are almost coincident, the curve falling away rapidly as the test proceeds. The forms of the curves given by tin and zinc persist in some alloys in which these constituents are present, notably gun metal and phosphor bronze, especially if the tin predominates. Brass, however, gives a rising curve (Fig. 184 (*b*)).

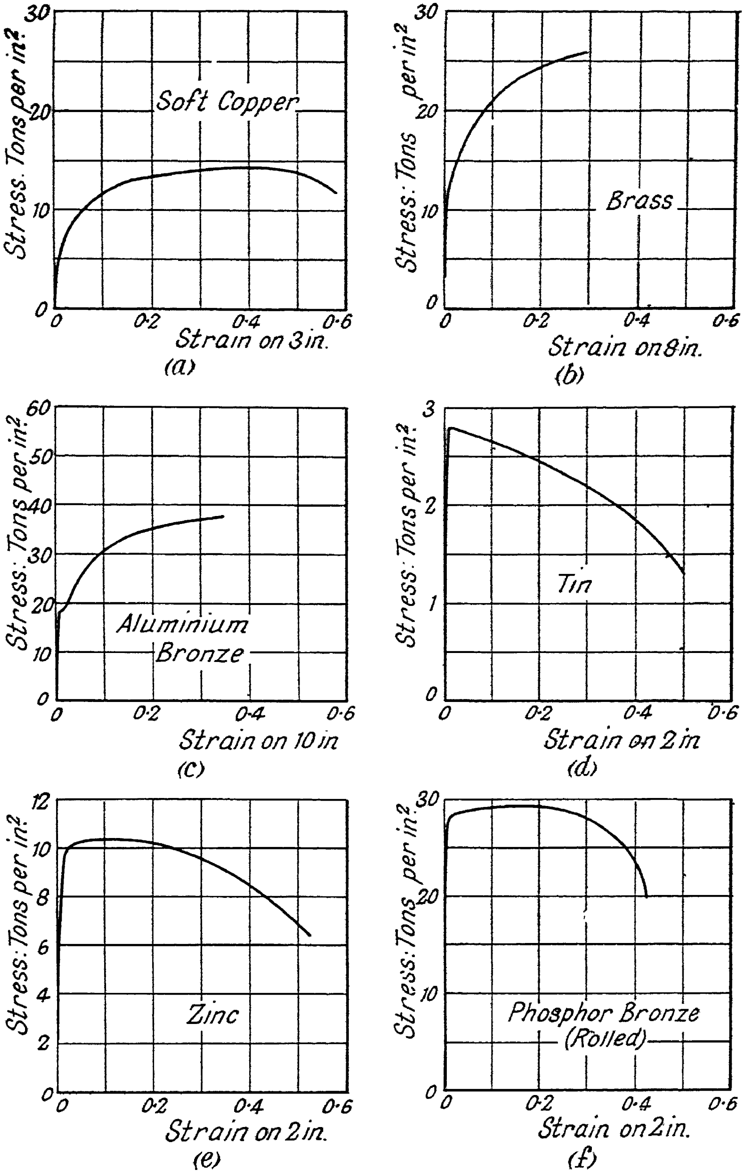


FIG. 184. STRESS-STRAIN DIAGRAMS GIVEN BY NON-FERROUS MATERIALS

specimen partially recovered its elasticity and after twenty-one days had completely recovered it.

Mild heat treatment after unloading,* say by boiling for twenty minutes at 100° C., furthers the recovery of the elastic properties in a remarkable degree. This was first shown by Muir in 1899. It is thus seen that the general effect of overstrain is to raise the yield point and to reduce the limit of proportionality.

The results of some tests by the Author on a specimen of mild steel illustrate this point (Fig. 186).

Low carbon steels generally recover their elastic properties after a rest or after boiling, but alloy steels are lacking in this respect. The graph in Fig. 187 is from a test by Dalby on a

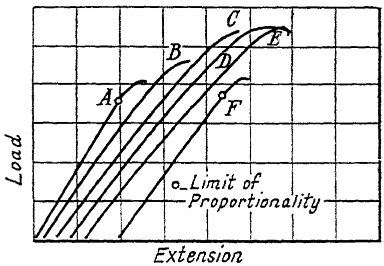


FIG. 187. RESULT OF TEST ON NICKEL STEEL

0·3 carbon, 3·68 nickel steel. Under the initial test, curve *A*, the limit of proportionality was 58 000 lb. per in.^{2} and the yield point 67 000 lb. per in.^{2} The material was stretched 2 per cent on a 5 in. gauge length and the second curve *B* shows that the limit of proportionality has vanished. Curve *C* was obtained after 6 per cent stretch, and curve *D* after a lapse of 24 hours, the bar having been turned down to a slightly smaller diameter. After boiling for one hour curve E was obtained, no recovery in elasticity being apparent. Finally the specimen was heated to 550° C. and kept at this temperature for 30 min. This treatment restored the elasticity. Both the limit of proportionality and the yield point were raised slightly as shown by the curve *F*.

Phenomenon of Strain Hardening. The phenomenon of hardening due to plastic distortion is termed *strain hardening.* The effect on a metal is to make it harder and less ductile. Such hardening disappears on annealing. A material which has been cold worked will exhibit tensile properties similar to

those of the overstrained material just discussed. Improvement in the tensile properties of a material in the direction of loading is not necessarily accompanied by improvement in its compression properties in the same direction.

If a single crystal of metal be tested in tension its strength will depend on the direction of the pull relative to the axes of the crystal. Plastic deformation of such specimens involves sliding or slipping along certain planes, and the commencement of slip depends on the shear stress along the plane and is independent of the normal stress. Increase of load increases the shear stress and the number of planes on which sliding occurs, with consequent further elongation of the specimen. The increase in stress necessary to continue the stretching represents the strain hardening of the crystal.

In a brittle material, fracture occurs due to the overcoming of cohesion on a certain crystallographic plane by the normal tensile stress. A commercial metal, on the other hand, exhibits the average effect on all the crystals of which it is composed. The result is that its mechanical properties are very nearly independent of direction.

Microscopic observation of a polished specimen under a tensile test shows a number of lines on the surface known as slip bands, due to the slipping of individual crystals under the action of the stress. The sliding stops at the crystal boundary. Some crystals may be less favourably situated than others to withstand tensile stress, and this is believed to be the cause of small deviations from Hooke's law in materials which are generally assumed to be elastic.

In ductile materials, especially those with a well defined yield point, considerable sliding along the planes of maximum shear takes place when the yield point is reached. These planes are indicated by the Lueder's lines which appear on the surface of a polished specimen an at angle of about 45° to the axis of the specimen. Individual crystals become strain hardened by distortion and on reloading the specimen the yield point is found to be higher than before.

Characteristic of Tensile Fracture in Ductile Materials. The strength of a test piece may be regarded as due to its resistance to sliding or to its resistance to separation. The associated types of failure that occur are termed *sliding failure* and *separation failure*, the former occurring in ductile and the latter in brittle materials. The relation between the two types of resistance

varies throughout a test. Resistance to sliding seems to increase as the velocity of deformation increases and decreases with rise of temperature. The resistance to separation is not affected to the same degree.

The type of fracture shown by a material depends on the conditions under which fracture takes place, as for instance whether or not plastic deformation due to sliding is prevented. The load extension diagram of a ductile material when flow is prevented by a groove in the test piece is similar to the curve (*b*), Fig. 181.

In a three-dimensional stress system such as that represented in Fig. 22, Chapter I, the maximum shearing stress is $(p_1 - p_2)/2$,

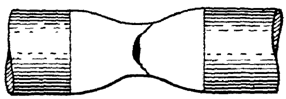


FIG. 188. LONGITUDINAL SECTION THROUGH FRACTURE IN A DUCTILE MATERIAL.

and if p_1 and p_2 be very nearly equal the maximum tensile stress may be many times the shear stress. Such conditions in a ductile material are productive of a brittle fracture.

In a specimen of mild steel under tension a three-dimensional stress condition obtains in the middle of the specimen. The metal at the neck is subject to tension in the radial as well as in the axial direction, and fracture takes place first by the formation of a crack in the centre of the section and then by yielding and sliding along planes inclined at 45° near the boundary. The result is the well-known cup-shaped fracture that appears when a bar of mild steel is tested to destruction in tension. Fig. 188 shows the appearance of the longitudinal section through a fractured test piece, the broken parts having been first fitted together and the metal in the neighbourhood of the break filed away.

On account of the deformation only the inclined portions at the boundary make contact, leaving a hollow space in the centre of the break. The diagram given by Professor Haigh to represent the state of affairs is shown in Fig. 189. On the little cube of material at the centre there is no tendency to make the metal shear, while the inner part of the neck has to pull in all directions in order to keep the curved outer fibres in equilibrium.

Hysteresis. Experiment shows that the elongation of a test-piece does not immediately follow the application of the load and that the specimen continues to elongate when the load has reached its final value. Tests made on single crystals show that if the specimen be loaded quickly according to the load extension line *oa*, Fig. 190, a lowering of temperature will occur due to increase in volume. If the application of the load be sufficiently rapid the process may be regarded as adiabatic, that is, there is no exchange of heat between the bar and the surrounding medium. The bar, however, gradually warms up and a slight additional elongation takes place represented by *ab*, the load remaining constant throughout this change. Rapid unloading gives the line *bc*, a rise in temperature occurring because of the decrease in volume. On cooling, the bar contracts by the amount *co* and resumes its original state. The line *oa* gives the adiabatic modulus of elasticity while the line *ob* would give the isothermal or constant temperature modulus.

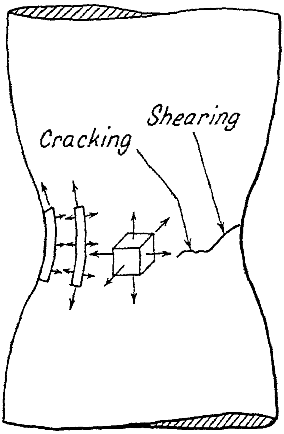


FIG. 189. FRACTURE OF A DUCTILE MATERIAL (*Haigh*)

In actual materials this time effect is much greater than in the single crystal and cannot be explained by purely thermo-dynamic considerations. One explanation is that it is due to the continued sliding within unfavourably situated crystals. The time effect after unloading is explained as due to the residual stresses which continue to produce sliding in crystals unfavourably orientated, thus causing creep in the material after the load is removed.

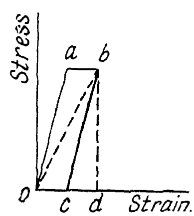


FIG. 190. ELASTIC HYSTERESIS

Many years ago Lord Kelvin gave the ratios of the quick to the slow Young's moduli for several materials, among others—

Iron . .	1·0026
Copper .	1·00325
Tin . .	1·0036
Zinc . .	1·008

The area enclosed by the loop in the diagram, Fig. 190, represents the dissipation of a certain amount of energy during the cycle of operations. With sufficiently delicate methods of measurement it is found that similar phenomena occur with metals generally, the strain lagging behind the stress even with stresses much below the value usually taken as representing the elastic limit of the metal. If a test-piece be loaded and unloaded repeatedly with tensile and compressive loads of the same final magnitude the resulting stress-strain diagram will take the form of a loop, Fig. 191. The shape of the loop formed

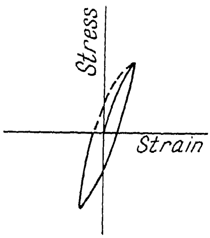


FIG. 191. ELASTIC HYSTERESIS LOOP

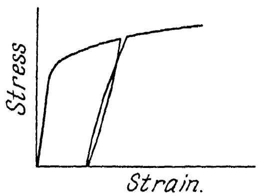


FIG. 192. HYSTERESIS LOOP OBTAINED WHEN LOADING AND UNLOADING BEYOND THE ELASTIC LIMIT

after the initial loading and unloading will depend on the history of the specimen. The size of the loop, termed the *hysteresis loop,* will depend on the material and on the range of stress. This is *elastic hysteresis.*

If a metal be stretched beyond the elastic limit the recovery of elasticity on unloading is usually incomplete, and on reloading, the loading line forms a loop with the previous unloading line. This hysteresis loop is much larger than the hysteresis loop within the elastic limit (Fig. 192). The phenomenon, so far, has not been completely explained.

Stress-strain Curves for Metals in Compression. Stress-strain curves for metals in compression exhibit as much divergence as do those for metals in tension.

The compression curve for cast iron, Fig. 193 (*a*), is similar in form to the curve for tension. Wrought iron, Fig. 193 (*b*), and mild steel, Fig. 193 (*c*), show a yield point, but less well defined than in tension and from this point onwards the curve rises continually. In this region the stress plotted is the nominal

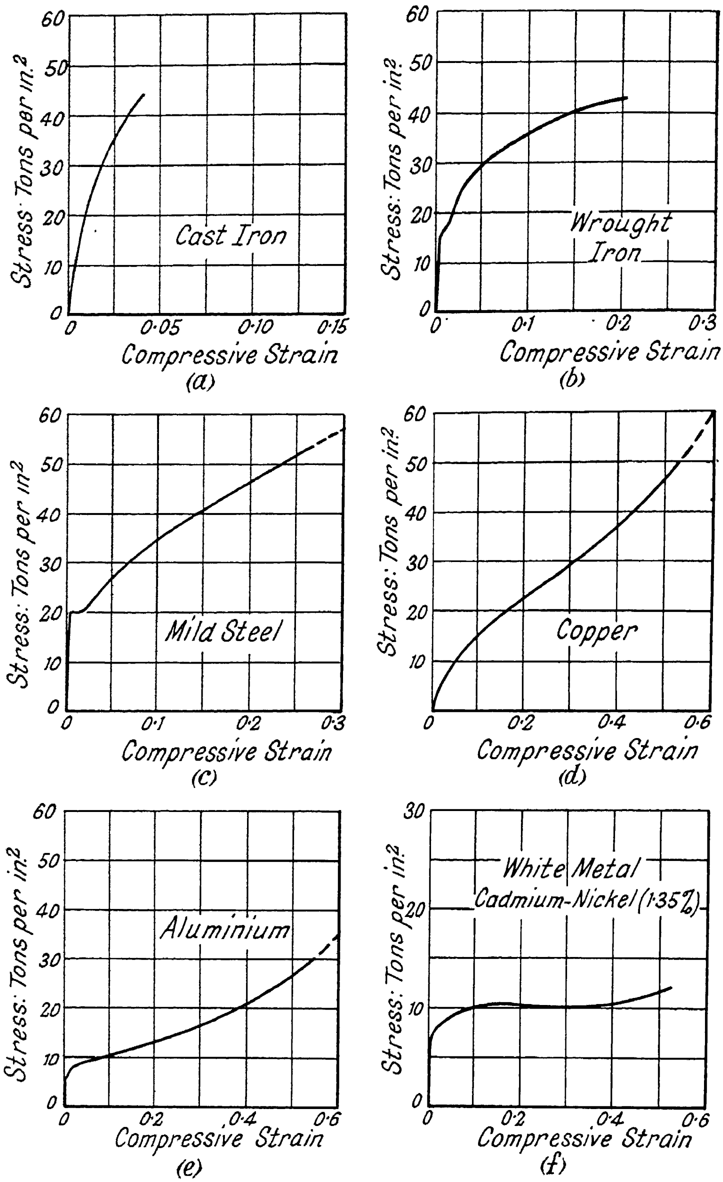


FIG. 193. STRESS-STRAIN CURVES OBTAINED IN COMPRESSION TESTS

value (load/original area), which is greater than the true stress owing to the spreading of the test piece. The curves for copper and aluminium are similar in form, with the yield better defined in the case of aluminium. The curve, Fig. 193 (*f*), is from a test on a cadmium-nickel alloy.

Failure of Ductile Materials in Compression. A short cylindrical specimen of wrought iron under compression will flatten out as shown in Fig. 194 and when about half its original length will develop longitudinal cracks which widen as the load is raised. Friction at the compression plates tends to prevent

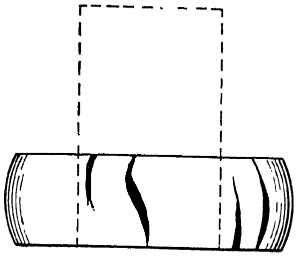


FIG. 194. CYLINDRICAL SPECIMEN OF WROUGHT IRON AFTER COMPRESSION TO ABOUT HALF THE ORIGINAL LENGTH

lateral spread of ends of the specimen. Mild steel takes up the same form but without cracking. The ultimate crushing strength is difficult to ascertain precisely. If the metal reached a perfectly plastic state the stress at which the material would flow would be constant. If l and l' and a and a' be the respective lengths and cross-sectional areas, then if no change of volume takes place

$$la = l'a'$$

and the real final stress

$$= \frac{\text{load}}{a'} = \frac{\text{load}}{a(l/l')} = \frac{\text{load} \times l'}{\text{volume of test piece}}$$

or,

$$\text{pressure of plastic flow} = \frac{\text{load}}{\text{volume}}\,(l - \text{reduction in } l)$$

It follows that if loads be plotted against extensions the resulting curve should be a rectangular hyperbola as the product is constant. It will be noticed from Fig. 194 that the curves for copper and aluminium in the later stage approach the hyperbolic form.

Failure of Brittle Materials in Compression. A brittle material fails in a different way, a short cast iron specimen, for instance, failing after the manner shown in Fig. 195. In a specimen under direct compressive stress p, the shear stress on a section inclined at θ to the axis of loading will be

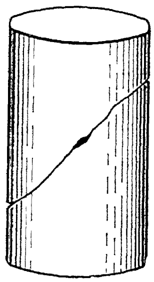


FIG. 195. MODE OF FAILURE OF A BRITTLE MATERIAL UNDER COMPRESSION

$$p \cos \theta \sin \theta$$

which reaches a maximum value when $\theta = 45°$.

With short specimens, instead of fracture occurring only on a single plane, the piece fails on a number of shear planes and crumbles up. Tests in which the angle of inclination of the plane of the resulting fracture can be measured are found, usually, to have an angle differing considerably from 45°. It is supposed that the normal pressure between the surfaces of separation increases the resistance to sliding. Navier's theory supposes the effect to be akin to friction. If q be the true shear resistance between the surfaces when there is no normal stress, then if r be the normal stress on the section the shearing resistance may be supposed to be expressible in the form

$$f = q - \mu r$$

where q is a shear stress and r a normal stress.

But $$q = p \sin \theta \cos \theta$$

and $$r = p \sin^2 \theta$$

where p is the direct compressive stress applied.

Hence $$f = p(\cos \theta \sin \theta - \mu \sin^2 \theta)$$

Fracture will occur where f is a maximum, that is, where

$$\cos^2 \theta - \sin^2 \theta = 2\mu \cos \theta \sin \theta$$

that is $$\mu = \cot 2\theta$$

If ϕ is the angle of friction, $\tan \phi = \cot 2\theta$, hence $2\theta + \phi = \pi/2$ and $\theta = \pi/4 - \phi/2$.

Some tests on cast iron gave $\mu = 0{\cdot}404$ to $0{\cdot}675$ with f varying from 10 to 16 tons per in.^{2}

Influence of Temperature on Mechanical Properties of Metals. The mechanical properties of metals are severely modified by

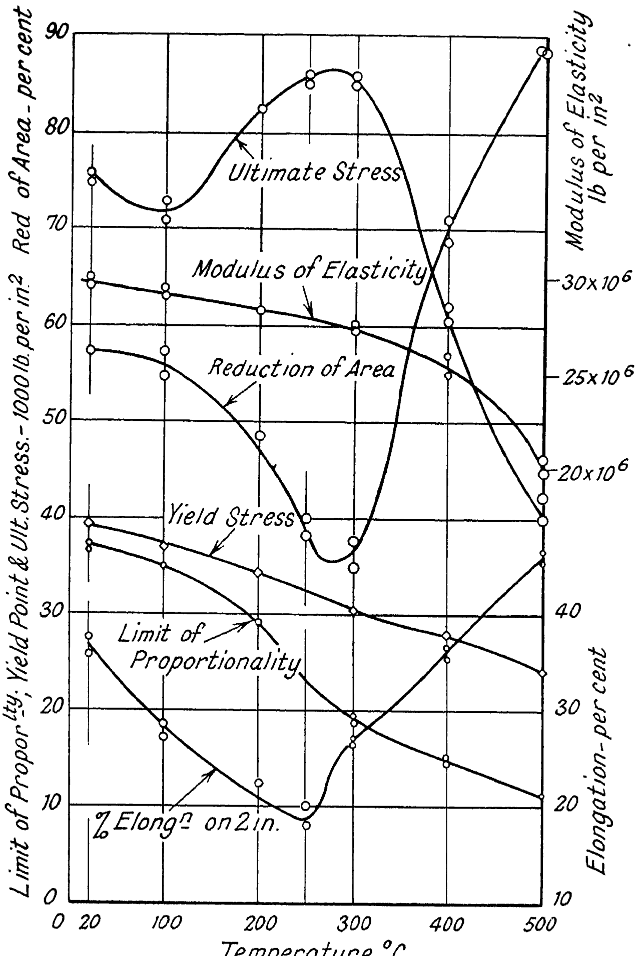


FIG. 196. TENSILE TESTS AT HIGH TEMPERATURE ON MEDIUM CARBON STEEL

(0·37 C., 0·63 Mn., 0·11 Si.)

(*Institution of Mechanical Engineers*)

rise of temperature when this exceeds about 200° C. The tensile strength of mild steel increases by some 30 per cent up to about 300° C. and then falls considerably as the higher temperatures

The strength of cast iron increases up to about 500° C. and then diminishes.

The results of some torsion tests on mild steel are given in Fig. 198.

Impact tests of steel show a decrease in the Izod or Charpy value up to 500° C., after which the values increase. Fig. 199

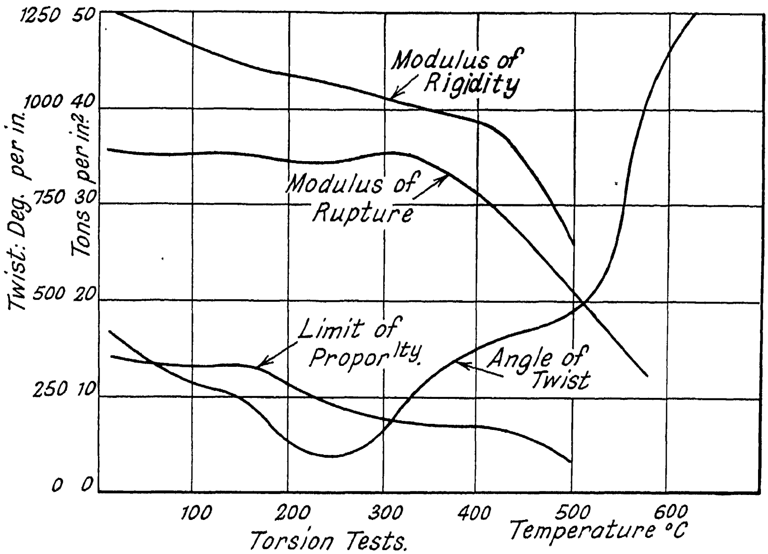


FIG. 198. RESULTS OF TORSION TESTS OF MILD STEEL AT HIGH TEMPERATURES

shows the results of tests on medium carbon steel (curve A) and on mild steel (curve B).

Hardness tests show irregularities over a temperature range of 0° C. to 700° C., the hardness numbers rising and falling throughout this range.

Fatigue strengths appear to reach a maximum in the neighbourhood of 250° C., but some steels, such as nickel-chrome, do not exhibit this increase.

Reference should be made to the Reports mentioned at the end of the book, where detailed information relating to the above tests will be found.

The study of the behaviour of metals at high temperatures has become of prime importance owing largely to developments

in high pressure steam plants, and intensive research is now being pursued.

At high temperatures the duration of the test has a marked

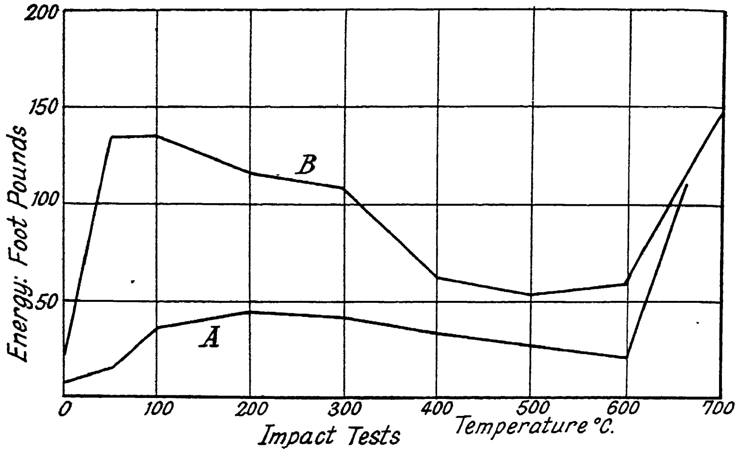


FIG. 199. IMPACT TESTS ON STEEL AT HIGH TEMPERATURES

influence on the results obtained, the load needed to produce fracture becoming smaller as the duration of the test is increased.

On the experimental side specimens are submitted to a constant load and temperature and the progressive creep under these conditions is investigated. If the results are plotted as an extension-time diagram a curve similar to that in Fig. 200 is obtained. The portion OA represents the initial extension. The rate of extension increases rapidly at first, but after the state represented by point A is reached it remains practically constant over a range AB. After the state corresponding with B is reached the rate of extension increases and fracture ultimately occurs. The life of the test piece lies within the range AB. If the stress be reduced the slope of AB decreases, but there appears to be no limiting creep stress at which the test piece can resist stress and high temperature indefinitely.

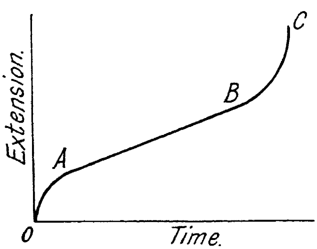


FIG. 200. EXTENSION-TIME DIAGRAM

Two effects come into play in such tests—

(1) Hardening of the metal due to plastic strain.

(2) Softening of the metal due to the prolonged action of high temperature.

In the extension-time curve, Fig. 200, the reduction in the rate of extension over the portion *OA* of the curve is due to strain hardening. The constant rate of extension over the range *AB* is brought about by the removal of the strain hardening produced by creep by the softening effect of the high temperature.

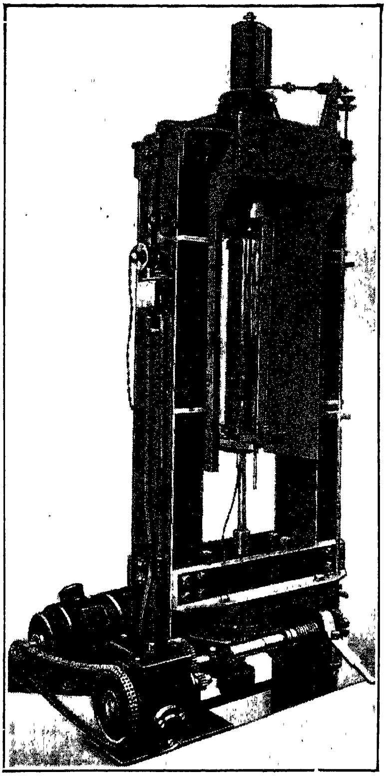


FIG. 201. METROVICK SLOW TENSILE TESTING EQUIPMENT
(*Metropolitan-Vickers Co. Ltd.*)

The Metrovick Slow Tensile Testing Equipment. The Metropolitan-Vickers Company have developed both slow tensile testing and creep testing equipment. The slow tensile testing machine is shown in Fig. 201. The load is applied through a train of gearing which provides rates of strain of 0·001 and 0·25 strain per minute as desired.

The specimen is connected with the loading mechanism through a short length of steel rod and a special chuck coupling which is visible in the illustration above the lower cross member of the machine. This simplifies the disconnection of the motor-driven loading mechanism and the connection of the loading gear that is fitted when it is desired to use the machine for creep testing. Load and extension are both recorded automatically by a photographic method, but direct visual measurement may be made. The temperature of the specimen can be finely adjusted up to

a maximum of 900° C. and is controlled automatically to within very close limits with a high degree of uniformity throughout the length of the specimen.

The slow tensile testing equipment serves a useful function in that it enables a range of steels to be placed in relative order of their creep resistance. Research has shown that for constant rates of strain the ultimate tensile strength will vary considerably with the temperature, and conversely, with constant temperature it will vary with the rate of strain. There is, in fact, a particular rate of strain which gives the optimum relation between the thermal hardening and the rate of increase of load and results in a maximum tensile strength for that temperature.

Generally, steel parts subjected to stress at elevated temperatures will operate for long periods; and thermal hardening and softening phenomena will not play any important part in their behaviour in service as these phenomena will take place during a short initial load period which (with the extremely slow rates of straining common under practical conditions) is negligible in relation to the time the part is in service.

Tensile tests, therefore, which are intended to indicate the relative creep resistance of materials under service conditions should be carried out in such a way as to ensure freedom from serious interference by thermal hardening effects, this requirement being met by testing at a suitably high temperature. The standard test temperature adopted by the Metropolitan-Vickers Company is 550° C. where the material represented by the specimen is to be used up to 500° C. When the service temperature is above 500° C. special precautions have to be taken against scaling of the specimen, and a test temperature of 550° C. or the service temperature, whichever is the higher, is employed.

It has been found that the slower rates of straining provide a better discrimination between steels, and the selection of a standard rate of straining resolves itself into the choice of a rate that will permit of a test being completed in a working day.

The results of some tensile tests on steels under various rates of straining are given in Fig. 202. A suitable rate of testing is 0·001 strain per minute.

The autographic recording apparatus consists of a drum inside the cylinder shown on the top of the machine in Fig. 201, and which is rotated through a vertical shaft driven from the

loading mechanism. The rotation is proportional to the extension of the test piece.

In one side of the otherwise light-proof cylinder surrounding the drum is a vertical slit, along which is fitted a glass tube of square section containing mercury and connected at the bottom to a metal reservoir. The base of the reservoir consists of a

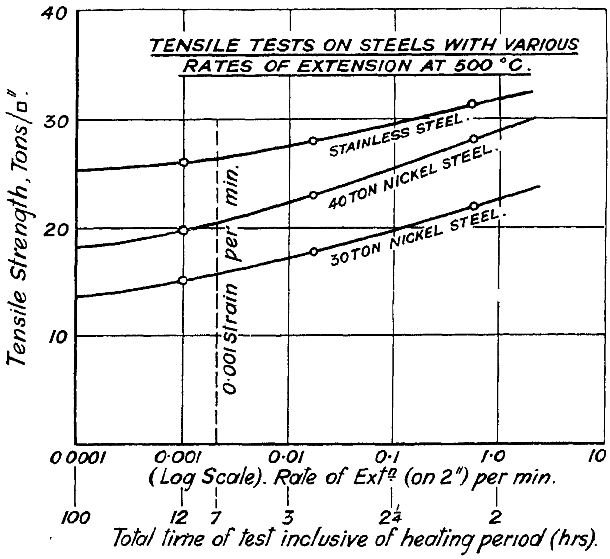


FIG. 202. TENSILE TESTS ON STEELS WITH VARIOUS RATES OF EXTENSION AT 500° C.
(*Metropolitan-Vickers Co. Ltd.*)

heavy diaphragm connected to the top end of the specimen. The deflection of the diaphragm as the load comes on the specimen varies the height of the mercury column in the tube.

A beam of light is projected through the slit in the cylinder on to the sensitized paper fitted to the recording drum, the mercury column acting as a shutter to vary the depth of the exposed portion of the record in accordance with the load on the test piece. A typical stress-strain diagram obtained with this apparatus is shown in Fig. 203.

The maximum stress exerted with the standard machine is 45 tons per in.^{2} on 0·1 in.^{2} of section. Specimens up to 8 in. gauge length can be accommodated.

Creep Tests. With regard to creep tests it may be remarked that tests of which the results have been published involve a duration which is not comparable with that of the service life of the material used in steam plants. Hence, in this respect, even tests of the longest duration can only be regarded as short-time tests.

Further, the majority of such tests have been carried out at stresses higher than working stresses, since most creep testing

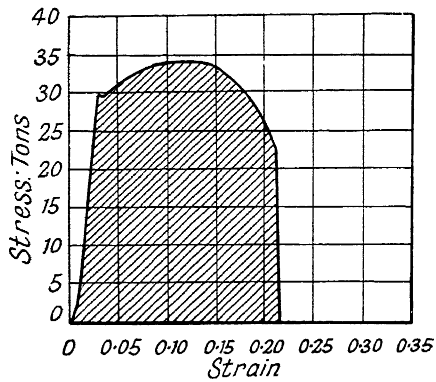


FIG. 203. AUTOGRAPHIC STRESS-STRAIN DIAGRAM MADE BY THE METROVICK PHOTOGRAPHIC RECORDING APPARATUS
(*Metropolitan-Vickers Co. Ltd.*)

machines are not capable of measuring in a reasonable time the slow rates of creep permissible under service conditions.

Practical requirements necessitate the measurement of rates of strain of 10^{-8} per hour within a reasonable time. The essentials for such apparatus are difficult of achievement. They are—

(*a*) Means for adjustment and measurement of the actual temperature of the specimen within 1° or 2° C.

(*b*) Means for obtaining constancy of temperature over long periods to within $\frac{1}{2}$° C. at all temperatures up to 850° C.

(*c*) Means for obtaining uniform temperature along the specimen within 1° or 2° C.

(*d*) Accurate and constant loading arrangements.

(*e*) Extensometer equipment capable of reading strains of 10^{-6} directly.

Metrovick Single-Unit Creep Testing Equipment. The most recent design of Metrovick single-unit creep testing equipment operating on a.c. supply is shown in Fig. 204. It is capable of

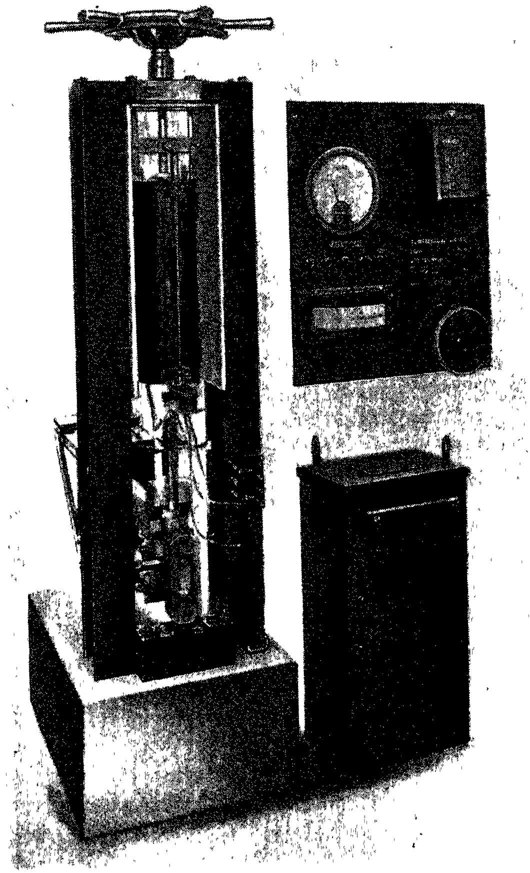


FIG. 204. METROVICK SINGLE UNIT CREEP TESTING EQUIPMENT
(Metropolitan-Vickers Co. Ltd.)

continuous service at temperatures up to a maximum of 700° C. The specimen temperature is thermostatically controlled to within 1° C. The design provides for a stress range of 0·5 to 20 tons per in.^{2} on a test piece having a 5 in. gauge length

and a cross-sectional area of 0·1 in.^{2} When using the dial indicator which is fitted to the end of the loading lever creep rates down to 10^{-5} strain per hour can be measured. For greater accuracy mirror extensometers must be used.

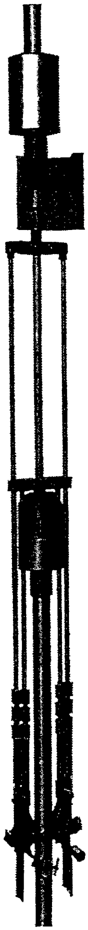


Fig. 205
Extensometer for High Temperature Measurements
(Metropolitan-Vickers Co. Ltd.)

Provision is made for lowering the load on to the test piece without shock at the commencement of a test. After switching on the current, the furnace heats up to the selected temperature at which the thermostat operates and controls the temperature to within the limits stated.

The equipment includes apparatus for the determination of proportional limits at elevated temperatures.

The furnace and the extensometer introduced by the Metropolitan-Vickers Company both differ from the conventional types. In the construction of the furnace a steel tube is used in place of the usual silica tube, and the extensometer is secured to the specimen by means of split clamps screwed together over specially enlarged portions at either end of the gauge length. For long-time high temperature tests experience has demonstrated the superiority of this method over the usual pointed-screw or knife-edge method of fixing. The instrument is shown in Fig. 205. Two mirrors with telescopes and scales are employed and the strain on an 8 in. gauge length may be read to $0{\cdot}25 \times 10^{-7}$.

Making allowance for some slight stagger of the plotted points it has been found in practice that constant creep rates of 10^{-8} strain per hour can be determined with reasonable certainty in from 400 to 500 hours. Fig. 206 shows a typical curve.

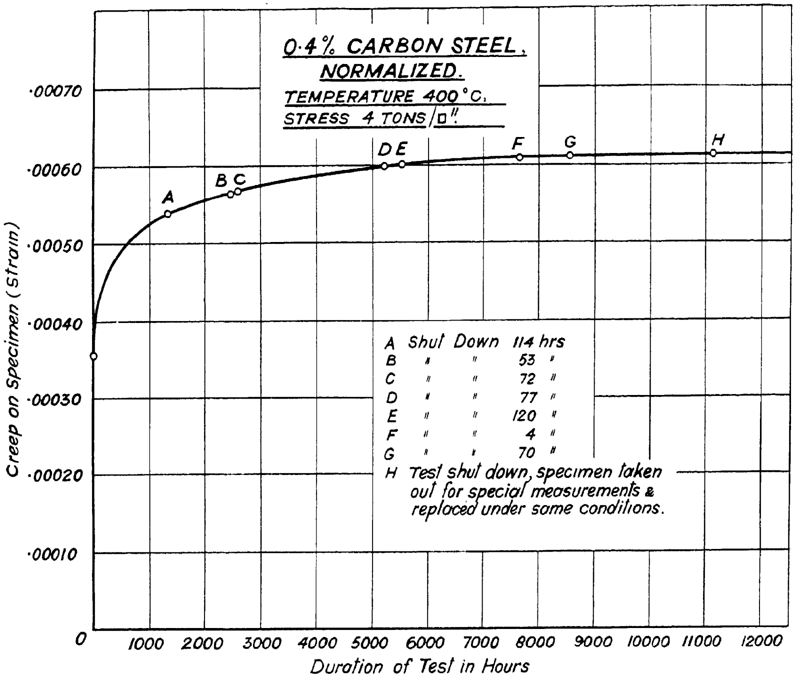


FIG. 206. TYPICAL CURVE SHOWING NATURE OF TESTING PERFORMED WITH METROVICK CREEP TESTING EQUIPMENT

0·4 per cent carbon steel normalized. Temperature 400° C., stress 4 tons/in.²

(*Metropolitan-Vickers Co. Ltd.*)

TABLES OF PROPERTIES OF METALS AND ALLOYS

TABLE XXII

Mechanical Properties of Some Metals and Alloys

Material	Maximum Stress in Tension lb./in.²	Yield Point Elastic Limit (E.L.) Limit of Proportionality (L.P.) lb./in.²	Elongation per Cent on 2 in.	Reduction of Area per Cent	Brinell Hardness Number	Impact Value C = Charpy I = Izod. ft.-lb.	Fatigue Limit (Rotating Beam Method) lb./in.²	Remarks
Cast iron	12 000 to 40 000	—	negligible	—	100 to 200	—	—	Strength in compression, 50 000–120 000 lb./in.² Strength in shear, 8 000–12 000 lb./in.² Brinell No. for chilled irons, 400–500.
	31 600	—	—	—	148	—	11 000	C 3·44, Mn 0·62, Si 1·1, P 0·51, S 0·09 (H. F. Moore)
Armco iron	42 000	15 300	50	80	—	—	28 200	(N.P.L.)
Carbon steel	63 800	39 200	37	64·5	125	65(I)	26 900	C 0·1, Mn 0·72, Si 0·06, normalized (Hatfield)
	77 300	46 300	30·2	54·6	—	32(I)	29 200	C 0·3, Mn 0·67, Si 0·14, Ni 0·21, normalized (Hatfield)
	99 000	62 800	24	43·5	—	18(I)	38 100	C 0·5, Mn 0·70, Si 0·18, Ni 0·10, normalized (Hatfield)
	115 000	67 700	23	40	227	3·3(C)	56 000	C 0·93, Mn 0·38, Si 0·13, P 0·019, heat treated
	188 000	97 200 (L.P.)	10	29	380	4·4(C)	97 500	
Alloy steel— 3½% nickel	130 000	—	18	50	270	35(I)	—	C 0·35–0·45, Mn 0·5–0·8, Cr 0·3, hardened and tempered
Ni-Cr-Mo	224 000	—	12	25	444	15(I)	—	C 0·3, Mn 0·45, Ni 3·95, Cr 1·25, Mo 1·25, hardened and tempered
3·6% nickel	154 200	142 000	15	55	—	—	74 000	McAdam
Cr-Vanadium	201 000	164 400 (E.L.)	13·3	53·5	—	—	94 500	C 0·55, Cr 0·99, Vd 0·19
Si-Mn spring steel	157 000	100 000 (E.L.)	16·5	40	—	—	62 000	

TABLE XXII (*contd.*)

MECHANICAL PROPERTIES OF SOME METALS AND ALLOYS

Material	Maximum Stress in Tension lb./in.^{2}	Yield Point Elastic Limit (E.L.) Limit of Proportionality (L.P.) lb./in.^{2}	Elongation per Cent on 2 in.	Reduction of Area per Cent	Brinell Hardness Number	Impact Value C = Charpy I = Izod, ft.-lb.	Fatigue Limit (Rotating Beam Method) lb./in.^{2}	Remarks
Copper	32 200	3 240(L.P.)	56	72	47	30·5(C)	10 100	
Hard-drawn wire	58 000	—	—	—	—	—	—	
70/30 brass	73 200	—	20	46	—	—	17 500	Cold rolled
Monel metal	100 400	53 300(L.P.)	31·8	67·5	—	—	40 000	Cold rolled, annealed (McAdam)
95/5 bronze	85 100	59 900(L.P.)	11·7	69	166	—	27 000	Cold drawn after annealing
Nickel	72 500	18 000 12 500(L.P.)	52	—	—	—	29 100	
Aluminium	22 600	11 300	16	65	45	—	10 500	Rolled
Duralumin	64 200	40 300 36 300(E.L.)	18·5	36·4	114	24	12 800*	Forged and heat treated. *Average of five tests by R. R. Moore.
Y-Alloy	58 000	40 300 33 800	21·5	32	114	15·4		Forged and heat treated
70% nickel alloy	83 500	56 000 35 000(L.P.)	46	70	—	187	35 700*	Ni 70·27, Cu 28·64. *Direct alternating stress (N.P.L.)
Ceralumin "C"	54 000	50 000 0·1% proof stress	1	—	135	—	18 500	Chill cast. Cu 2·5, Ni 1·5, Mg 0·8, Fe 1·2, Si 1·2, Cerium 0·15. Brit. Pat. No. 403700.

TABLE XXIII

Values of Young's Modulus and Modulus of Rigidity

Material	Young's Modulus (E) (lb./in.^{2})	Modulus of Rigidity (G) (lb./in.^{2})
Cast iron	12×10^6 to 23×10^6	5×10^6 to $7\cdot5 \times 10^6$
Wrought iron—		
Bar	29×10^6	$10\cdot5 \times 10^6$
Plate	26×10^6	14×10^6
Carbon steel	28×10^6 to 31×10^6	11×10^6 to 13×10^6
3 per cent nickel steel	28×10^6	—
30 per cent nickel steel	23×10^6	—
Nickel	$29\cdot5 \times 10^6$	11×10^6
Copper	15×10^6	—
Hard-drawn wire	18×10^6	7×10^6
Brass—		
Hard drawn and cast	12×10^6 to 16×10^6	5×10^6
Phosphor bronze	15×10^6 to $17\cdot5 \times 10^6$	6×10^6 to 7×10^6
Monel metal	26×10^6	10×10^6
Y-Alloy	10×10^6	—
70 per cent nickel alloy	26×10^6	10×10^6
Aluminium	9×10^6	$3\cdot8 \times 10^6$
Duralumin, heat treated	10×10^6	$3\cdot5 \times 10^6$

TABLE XXIV

Weight of Metals

	Weight per in.^{3} (lb.)
Aluminium—	
Sheet	0·096
Cast	0·092
Aluminium bronze	0·275
Brass—	
Cast	0·301
Wire	0·307
Copper	
Cast and sheet	0·317
Wire	0·321
Duralumin	0·101
German silver	0·300
Gunmetal, 90 per cent Cu.	0·306
Iron—	
Cast	0·260
Wrought	0·280
Lead	0·410
Monel metal	0·319
Nickel	0·318
Phosphor bronze—	
Cast	0·310
Steel, average	0·282
Tin	0·262
Zinc—	
Cast	0·247
Rolled	0·260

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